THE
MATHEMATICAL
GAZETTE

EDITED BY

W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF

F. S. MACAULAY, M.A., D.Sc.

AND

PROF. E. T. WHITTAKER, M.A., F.R.S.

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Change of address should be notified to a Secretary.

If Copies of the Gazette fail to reach a member in consequence of a change of address, of which the Secretaries have not been informed, duplicate copies cannot be supplied.

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NOTICE.

The following Reports have been issued by the Association:—(i) "Revised Report on the Teaching of Elementary Algebra and Numerical Trigonometry" (1911), price 3d. net; (ii) "Report on the Correlation of Mathematical and Science Teaching," by a Joint Committee of the Mathematical Association and the Association of Public School Science Masters, price 6d. net; (iii) A General Mathematical Syllabus for Non-Specialists in Public Schools, price 2d. net. These reports may be obtained from the Publishers of the *Gazette*.

(iv) **Catalogue of current Mathematical Journals**, with the names of the Libraries where they may be found. Pp. 40. Price, 2s. 6d. net.

"Even a superficial study convinces the reader of the general completeness of the catalogue, and of the marvellous care and labour which have gone to its compilation."—*Science Progress*, Jan. 1916.

(v) **Report of the Girls' Schools Committee, 1916**: Elementary Mathematics in Girls' Schools. Pp. 26. 1s. net.

(vi) **Report on the Teaching of Mechanics, 1918** (*Mathematical Gazette*, No. 137). 1s. 6d. net.

(vii) **Report on the Teaching of Mathematics in Public and Secondary Schools, 1919** (*Mathematical Gazette*, No. 143). 2s. net.

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The Mathematical Association.

President: PROFESSOR E. T. WHITTAKER,
M.A., Sc.D., F.R.S.

THE Annual Meeting of the Mathematical Association was held at the London Day Training College, Southampton Row, London, W.C. 1, on Wednesday, 7th January, 1920, at 5.30 p.m., and Thursday, 8th January, at 10.0 a.m. and 2.30 p.m.

WEDNESDAY, 5.30 *p.m.*

(1) **ADVANCED SECTION**: "A survey of the numerical methods for solving Equations," by the President.*

> The Lobachefsky-Graeffe method. The Ruffini-Horner method. The inverse Interpolation method. Newton's method with Dandelin's and Sang's improvements. Iterative methods. Gauss' method. Fürstenaus' method. Recent methods.

THURSDAY, 10 *a.m.*

(2) The following Report of the Council for the year 1919 was distributed, taken as read, and adopted:

During the year 1919, 86 new members have been elected, and the number of members now on the Roll is 722. Of these 7 are honorary members, 42 are life members by composition, 33 are life members under the old rule, and 640 are ordinary members. The number of associates is about 160.

The Council regret to have to record the deaths of Mr. W. E. Ashby (of Normanton Grammar School), Mr. P. E. B. Jourdain,

* We hope to publish this paper and the President's Address in a future number.—[ED.]

Mr. G. W. Palmer (of Christ's Hospital), and Dr. Artemas Martin (of the U.S. Coast and Geodetic Survey Office). Mr. Ashby was killed in action.*

By the untimely death of Mr. G. W. Palmer, Master of the Royal Mathematical School, Christ's Hospital, the teaching world has suffered an irreparable loss. His contributions to the work of the Teaching Committees were invaluable for their freshness and cogency.

A report on the Teaching of Mechanics was published in the *Mathematical Gazette*, No. 137, issued at the end of 1918.

A report on the Teaching of Mathematics in Public and Secondary Schools forms the contents of the *Mathematical Gazette*, No. 143, for December, 1919.

An election of the Teaching Committees will be held early in 1920, after an interval of six years.

Owing to the increased, and increasing, cost of printing, and the general rise in necessary expenditure, the Council recommend that the annual subscription to the Association be raised from 10s. to 15s. It is hoped that this will make it possible to issue, as formerly, six *Gazettes* each year, and to restore them to their former size. As a consequence of this increase in the annual subscription it will be proper to increase the Composition Fee for Life Membership from 7 guineas to 10 guineas.

Two vacancies on the Council have arisen owing to the death of Mr. G. W. Palmer and the resignation of Miss M. R. Baldwin. The members present at the annual meeting will be asked to nominate and elect two members to fill these vacancies.

The Council recommend that Professor G. H. Bryan, Sc.D., F.R.S., and Mr. Rawdon Levett, be elected Honorary Members of the Association. Mr. Levett was most active as a founder of the Association for Geometrical Teaching in 1871.

The Council again desire to acknowledge the indebtedness of the Association to Mr. Greenstreet for his services as Editor of the *Mathematical Gazette*, and to offer their thanks to the authorities of the London Day Training College for their kindness in affording accommodation for the Annual Meeting, and for the meetings of the Council and of the Committees which have been held during the year.

(3) The Treasurer made a short statement on the Finances of the Association.

His Report for the year 1919 was approved.

(4) Mr. Siddons spoke briefly on the work of the Teaching Committees.

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* We rejoice to take the opportunity of stating that the published report of the death of one of our members, Mr. J. Blaikie, is unfounded.

(5) The following changes in the Rules were proposed by the Council, and agreed to:

(i.) That owing to the greatly increased cost of printing, the subscription to the Association be increased to 15/- a year.

(ii). That Institutions and Societies, as such, be admissible to membership.

(iii). That the free issue of Reports be confined to members and contributors.

(iv.) That the Librarian be an official member of the Council.

It was agreed, on the motion of Prof. W. P. Milne, that the Annual Dinner be revived in Jan., 1921, and that a Summer Meeting be held at Leeds, about Whitsuntide, 1920.

(6) The Election of Officers and Council for the year 1920.

Two vacancies were caused by the death of Mr. G. W. Palmer and the resignation of Miss M. R. Baldwin.

Dr. W. F. Sheppard and Mr. R. C. Fawdry were elected to fill the two vacancies.

(7) **GENERAL SECTION.** "Geometry Teaching: the next Step," by Mr. C. Godfrey, M.V.O.

(8) "Convention and Duplexity in Elementary Mathematics," by Professor E. H. Neville.

(9) "The Place of Common Logarithms in Mathematical Training." A discussion was opened by Miss H. M. Cook.

THURSDAY, 2.30 *p.m.*

(10) The President's Address: "Some Mathematical Problems awaiting Solution."

(11) "The Teaching of Mechanics to Beginners." A discussion was opened by Mr. R. C. Fawdry.

GLEANINGS FAR AND NEAR.

38. The Intrusion of Mathematics.

The Governing Body of the Union shall be a Council elected by the branches. Each branch shall elect to the Council a number of members equal to the integer nearest to n/s, where n is the number of members of the branch and s is a number to be fixed from time to time by the Council. Zero shall not be counted as an integer. If n/s is of the form $p+\frac{1}{2}$, where p is an integer, it shall be taken as equal to $p+1$, according as p is odd or even.—National Union of Scientific Workers. Proposed Rule 8 (election of Governing Body), 1919.

[A journalistic comment was] Were the Registrar to pass the rule as it stands, of course, "'twould be recorded for a precedent, and many an error by the same example would rush into the State." He will therefore, it is to be feared, consider it necessary to nip such algebraic impertinence in the bud, as most "tolerable and not to be endured," and have it writ "plain English," should the heavens fall.

GEOMETRY TEACHING: THE NEXT STEP.

By C. Godfrey, M.V.O.

I draw your attention to two dates of some importance in the teaching of Geometry. In 1903 Cambridge ruled that "Any proof of a proposition will be accepted that appears to the examiners to form part of a systematic treatment of the subject"; in 1909 the Board of Education issued Circular 711. These two dates determine three periods of time.

Before 1902, Geometry teaching was ruled by Euclid's sequence; we may describe this as the ice-age; teaching was frozen. What we were compelled to teach was called Euclid, but of course it was not really Euclid, for we did not teach Book 5. It was Euclid with the backbone removed; Euclid filleted.

I suppose that most of us were at school then. Personally I have no complaint to make of the teaching I received in these conditions; perhaps I was unusually fortunate, for my teacher was Mr. R. Levett, a name very well known to members of this Association. Presumably everyone present in this room was very far from being an average mathematician at school; we were all gifted mathematicians, otherwise we should not now be teaching the subject. What suited us may not have suited the average boy; in fact, when it became my duty to teach Geometry, it was very obvious to me that the average boy could not be taught properly in such conditions as were then possible. No far-reaching reconstruction could be planned while we were restricted to Euclid's sequence. Well, that is all over now; we need not revive the controversies of the nineteenth century. But we must remember that similar controversies would come to life again if any attempt were made to impose any one fixed sequence on teachers; happily there seems to be no immediate prospect of any such proposal being accepted.

What was the Mathematical Association doing during this period? Under the name of the A.I.G.T., and afterwards as the M.A., it did much valuable work; it did all that was possible to alleviate the situation. But it must have been disheartening work, for no fundamental cure was feasible until the Universities released us from bondage. On the eve of our liberation the M.A. published a report on Geometry teaching; a very conservative report, as it was considered impracticable to secure the abolition of the sequence. This report became obsolete in 1902, and no report on Geometry has emanated from your Teaching Committee since that date. This may seem strange, but our inaction has not been due to slothfulness. We have felt that during the period of rapid change it was undesirable that the Association should attempt to crystallise teaching. Until matters have settled down, it is for individuals to urge new ideas; an Association cannot easily do more than register the average opinion of sound teachers; and at the present moment a report on these lines would delay rather than advance the movement of reform. I do not consider that the time has yet come for the Association to support with its authority any particular method of teaching Geometry.

The next period is that between 1903 and 1909. What was the immediate effect of the Cambridge decree? Cambridge gave liberty of sequence, and at the same time published a list of propositions, explicitly not in order of treatment; other Universities have taken similar action.

The effect seemed to us at the time to be very great; but I suppose that as a matter of fact a great majority of schools went on teaching in the old way, and no doubt many to this day have been quite unaffected by the new spirit. But in those schools which moved with the times the immediate effect was probably rather chaotic. This was quite natural and inevitable; if the former period may be compared to an ice-age, this was the break-up of the ice. We did not at once realise how to make the best use of our liberty.

One mistaken development was a great abuse of drawing and measurement. These methods have their proper and permanent place in the teaching of Geometry, but many of us did not understand at the outset their proper limitations. These limitations have been explained very clearly by Mr. Carson, to whom teachers of mathematics owe a great debt. He has insisted on the *rôle* of intuition in learning Geometry. Some geometrical properties are intuitively obvious to children: for instance it is obvious that vertically opposite angles are equal. Probably these intuitive perceptions are the result of numberless unconscious experiments of everyday life; they are lying in the mind, more or less hidden, ready to be brought into the light and expressed in clear language. It is a mistake to attack such properties as these by drawing and measurement; the result may possibly be a weakening of geometrical intuition; it certainly will be a waste of time, and will lead to boredom.

On the other hand, there are many other properties that are not obvious to intuition; consider the property connected with the product of the segments of chords of a circle through a fixed point. It is inappropriate to *approach* such properties by a purely logical method; they must be proved logically as soon as they are believed to be true, or at any rate likely; but it is not superfluous to secure assent by a preliminary experiment.

After we had fought our way for some years towards reconstruction, we received Circular 711 from the Board of Education, and the third period of which I speak is that dating from this publication to the present time. You are familiar with the proposals of Circular 711. Briefly is it suggested that we should recognise three stages in our teaching.

Stage 1. To be concerned with the fundamental concepts and generally entitled Introductory Practical Work. This stage will not take up very much time; possibly it will not take up more than a term.

Stage 2. To deal by intuitive methods with a few fundamental propositions. These propositions are the groups concerned with angles at a point, parallel lines, congruence of triangles, angles of triangle and polygon. The "angle at a point" theorems are too obvious to a boy of 12 for him to feel the need of a formal proof. The "parallel lines" propositions involve us in grave controversy as to sequence; by appealing to intuition we outflank these troubles. The congruence theorems are proved mainly by superposition. It is difficult for a young boy to understand what he is getting at. To his mind, the triangles fit clearly enough, and he does not see that he is proving anything substantial. In fact, is he proving anything? Hilbert found it necessary to assume Euclid 1-4 as an axiom, or at any rate to assume a part of the proposition; why should a boy of 13 be expected to prove what Hilbert failed to prove? We are to substitute for the group of congruence propositions a discussion of the data needed for the determination of a triangle; in this the boy finds something substantial and real.

The "angle-sum" theorems are not quite on the same footing; they are quite good propositions for a boy to prove; the general argument for taking them without formal proof is that they are involved, together with theorems of parallels, in the sequence difficulty. The proposals of 711 in this particular have not found universal acceptance, but after all this is a comparatively unimportant detail.

Stage 3 was to include the main part of the subject, the general deductive development; and it is with Stage 3 that I have to deal in the proposals I shall lay before you to-day.

On the whole, it may be said that the plan advocated by Circular 711 has been widely accepted in this country. This acceptance depends to an important degree on the assent of the examining bodies, especially as regards examinations for boys of 13 or 14. I believe that the starred propositions are omitted from the schedules governing the Common Entrance Examination

and the examination for Naval Cadetships. The same proposals are sanctioned in the mathematical curriculum drawn up by a joint committee of the H.M. Conference and the Association of Preparatory Schools; the syllabus covers the ages of 9-16. They are also accepted in the regulations of the Oxford Local Examinations, and it is to be expected that in the course of the next year or so the assent will be obtained of all the chief examining bodies whose activities affect boys of public and secondary schools; perhaps London University will lag behind. The Board of Education—or rather the Schools Eaminations Council, a body representing the Board in this matter—has recently conducted an investigation into the conduct of the seven examining bodies awarding School Certificates in England; the report of the mathematical investigators concurs with the main proposals of Circular 711.

This question then is approaching solution, and it now remains to discuss the next step. You may say, why is a further step necessary? What remains to be reformed? My quarrel is with Stage 3 of the circular. I must for a moment ask you to consider the two separate subjects of logic and geometry. Geometry has traditionally been represented in a dress of formal logic; or perhaps we may describe logic as the medicine and geometry as the jam. I have not studied logic, but I suspect that the particular form we meet with in geometry by no means exhausts the subject. Still, I am not prepared to say that we ought not to study this sort of logic; what I want to ask you is, at what age ought we to study it? Stage 3 will naturally be entered at about the age of 13; is there any natural break in the development of a child's mind which justifies a sudden step from the quite informal methods of Stage 2 to severely formal methods in Stage 3? I say that a boy of 13 is not ripe for the full rigour of the game: I am sure that he is not ready for it before the age of 15, and I expect that even 15 is too early. I should like to recast the work of Stage 3 and to relegate the strict Euclidean method to a Stage 4, which should certainly not be taken up before 15 or 16. The natural place for Stage 4 would be after the School Certificate age, *i.e.* after 16; but until the examining bodies have been converted—that is the next campaign—it may be necessary to attack Stage 4 a year or so before the School Certificate is taken.

The heart of my discourse is my new Stage 3. Now I want to be free at this stage to adopt whatever methods seem appropriate in each instance. There are many propositions which can perfectly well be dealt with by Euclidean methods in this stage; an instance is the group of propositions dealing with the angle properties of a circle. The things proved are striking and interesting; the boy feels that they are worth proving; there are no philosophical subtleties in the proofs. On the other hand, there are properties of such a character that the boy does not at this age appreciate the need of formal treatment. Of these I instance two classes: properties connected with Symmetry and properties connected with Similarity.

If any one will take the trouble to analyse the exercises in an ordinary text-book, he will find that a very high percentage require the proof of the congruence of two triangles. *And practically all of these exercises are proofs of properties depending on nothing but symmetry.* Now, in the first place, why do we not recognise explicitly in our teaching the existence of a fundamental geometrical fact like symmetry? As far as I know Euclid nowhere speaks of symmetry. Secondly, are not all these symmetry properties obvious to the boy? and is it not very tedious for him to spend all this time in proving the obvious? There comes a time in the development of the mathematician when he delights in proving, or disproving, the obvious; that time is certainly not before the age of 15. If you interrogate any of your non-mathematical friends, they will tell you that this continual proving of the obvious was one of the unpleasant features of their mathematical work at school; probably they gave up the study of mathematics some time before they left school, and this feature of the work was one of the causes that cooled their affection.

I would abolish all these congruence exercises. I would present to boys once and for all the two types of plane symmetry—Symmetry about a line and Symmetry about a centre—and after that I would use the principle of symmetry fearlessly. You will not find more than one standard proposition in which symmetry cannot be used to replace congruence; that proposition is the theorem of Pythagoras, and this can be proved much more shortly by similarity.

Traditionally similarity comes rather late in the book, and the reason for this is historical; Euclid could not deal with similarity to his satisfaction until he had developed the theory of incommensurables in Book 5. We do not teach the theory of incommensurables, and there is every reason for putting similarity quite early in the course. It is as easy and patent a fact as symmetry. A boy is appreciating the existence of similar figures whenever he makes a drawing from nature, whenever he takes a photograph, whenever he goes to a cinema. Similarity must come in quite early; but for this purpose it must be presented as a matter for intuition. There are three fundamental theorems connected with similar triangles, analogous to the three fundamental theorems of congruence. There can be no question of teaching at the age of 13 or 14 these fundamental theorems of similarity; their formal proofs must be postponed to Stage 4; but the facts involved can be made clear by intuitive methods.

Stage 3, then, is to cover the whole of elementary Plane Geometry (and some Solid Geometry, I hope) by eclectic methods—sometimes a formal proof, sometimes an appeal to intuition. The amended course may now be summarised—and I am persuaded that it is a natural and inevitable development of Circular 711, and by no means an antagonistic proposal.

Stage 1. Introductory practical work.

Stage 2. Presentation by intuitive methods of the fundamental propositions.

Stage 3. Plane and Solid Geometry, with free choice of method, sometimes informal and sometimes formal: what modern language teachers call *méthode mixte*.

Stage 4. Recapitulation of Plane Geometry, the attention being directed to a formal chain of propositions and the interest being logic rather than geometry.

You will notice that I am proposing to convert a 3-speed gear into a 4-speed gear: in practice the change from the second to the third speed in the old motor has been found too abrupt.

I will not apologise for discussing before an audience of practical teachers the bearing of examinations on this new method. The only examinations that restrict a school are external examinations. The external examinations that concern preparatory and public schools are the Common Entrance Examination and the School Certificate Examination. The managers of the C.E.E. are a progressive body, and it should not be difficult to convert them to a policy that is so unquestionably sound for boys of 13. The seven university bodies that grant School Certificates cannot be expected immediately to concur with new methods of teaching: naturally they follow rather than lead. For some years they will continue to examine on familiar lines; how will the course of teaching I have described fit in with these examinations? Clearly it will fit admirably; Stage 4 will be an excellent preparation for the formal part of these examinations. But I have not concealed my opinion that, ideally, Stage 4 should come after, not before, the School Certificate. As a matter of fact, I believe that a boy who has mastered my third stage and postponed Stage 4 has nothing to fear from the School Certificate ordeal. He may fail to write out some of the theorems set: but what he may lose on the swings he will gain on the roundabouts; no paper consists solely of

theorems, and no boy has to do the whole paper in order to pass with credit. I do not think that external examinations need deter an enterprising teacher from teaching as he thinks best; at any rate, his teaching will be alive, and the effect on his pupils' work will be unmistakable.

In the course of the subsequent discussion the following observations were made by Prof. T. P. NUNN.

Some who are present this morning will remember that at the corresponding meeting held in this room three years ago I gave an address on the subject—"The Curriculum in Geometry"—upon which we have just heard so interesting and valuable a paper. What I then said was not accessible to Mr. Godfrey, for I was remiss enough not to print even an abstract of my argument. It is, therefore, with all the more pleasure that I note a very close congruence between the recommendations we offer.

Like Mr. Godfrey, I divided the curriculum into four phases: (1) A preliminary stage, in which the learner acquires, under guidance, his fundamental spatial experiences and ideas. (2) An "intuitional" stage, in which he reduces those experiences and ideas to clearness, and is made aware of some of their more important connections and consequences. I referred to this stage comprehensively as "boy scout geometry," and again, like Mr. Godfrey, gave an important place in it to the principle of similarity. (3) A stage in which the essential theorems of geometry are derived deductively from a wide basis of assumptions by a free use of the principles of congruence, similarity, parallelism and symmetry. (Also, I should have added, of the principle of projection, which, as I showed by practical illustrations, could very profitably be used to enrich the scope of elementary geometry by inclusion of the salient properties of the conic sections.) (4) A stage of "strict" geometry, based upon a minimum number of carefully examined postulates.

Like Mr. Godfrey, I urged that Stage 4 should be postponed until after the age of 16 (*i.e.* until after the General School Examination), and that it should be made more rigorous and philosophical than would be suitable for the non-specialist. Mr. Godfrey seems to contemplate that arguments based on symmetry should not be reduced to written "propositions." Since my proposals imply that General School and Matriculation Examinations in geometry should be based on Stage 3, I assumed that our pupils *would* be required to reduce their arguments to written form. I see no difficulty here, for there are excellent models in several standard French geometries. I agreed with Mr. Godfrey in discarding from the earlier stages both the Euclidean proofs of the fundamental propositions with regard to congruence and similarity, and their modern substitutes. On the other hand, I suggested that there were proofs of these propositions which are quite appropriate to Stage 3, and are in a line with the modern rigorous treatment—more rigorous than Euclid's—to be aimed at in Stage 4. I may add that I have since developed the treatment I proposed for the conditions of similarity, and have worked it up into what seems to be a satisfactory shape. I also urged that tri-dimensional geometry should, from the outset and all through the course, receive a greatly more important place than at present. I contended that the principles of congruence, symmetry and similarity should be taught by the study of tri-dimensional as well as plane instances; the principle of symmetry, in particular, being introduced by means of models of right and left-handed houses, etc.

As far as we differ, it is mainly in matters of detail, not of principle. I conclude by re-emphasising my cordial agreement with the general tenour of Mr. Godfrey's proposals, and by expressing the hope that the Association will shortly take steps to get them incorporated in the practice of the schools and the syllabuses of public examinations.

CONVENTION AND DUPLEXITY IN ELEMENTARY MATHEMATICS.

BY PROF. E. H. NEVILLE, M.A.

THE position I wish this morning to urge you to adopt is adequately explained on the programme—it is that the President, who as a man has come some 400 miles South to these meetings, as a mathematician has travelled *both* $+400$ miles South *and* -400 miles North.

I anticipate—discussion presently will show if I am wrong—that I must face criticism of two mutually destructive kinds; that some of you will tell me that I am only preaching what everybody else is busy practising, while others will say it is of little comfort to the pupil that Prof. Milne has made Calculus easy if I am to be allowed to make Coordinate Geometry hard.

To the first comment I can only say that I have found nothing in current English books to suggest that the duplexity of vectors is a commonplace. To put in the pillory the many mathematicians—some of them prominent members of this Association—whose writings I have examined on this point, would be a long and ungracious task, and I have no desire to make the attempt; it will be sufficient if I refer to two books and ask you to accept them as typical. Dr. Silberstein's *Vectorial Mechanics* is designed to introduce vectors to those to whom they are unfamiliar, and there is no attempt to secure brevity in the early stages by short cuts or assumptions to be made in practice, nor is the author a man to be bound unduly by other people's conventions. Dr. Livens' *Electricity and Magnetism* begins with a summary of vectorial definitions and symbols, not as an exposition, but for convenience of reference, and here if anywhere we may hope to find the view that is to be regarded as orthodox. Both Silberstein and Livens say explicitly that the tensor of a vector is essentially positive, and speak of *the* direction of a vector.

To rebut the opposite accusation in detail would leave me no time for generalities. But after all there is no new conception to be imparted to the student. Before position can be specified on a line through an origin O, the line is given a definite sense, and then the distances from O of points on one side of O are positive, while the distances from O of the points on the other side of O are negative; if the sense is changed, the distances that have been positive become negative. Which is the more natural, to say that the notion of sense belongs only to the coordinate axes, or to see it involved in every question of measurement? As to subsequent definitions and proofs, the alterations necessary are trivial. For example, if ρ, σ are two vectors having amounts r, s, and ϵ is an angle between the direction in which ρ has the amount r and the direction in which σ has the amount s, then $rs \cos \epsilon$ depends only on the vectors themselves, not on the choice of directions of measurement, for to reverse the direction of measurement of ρ is simultaneously to change the sign of r and to alter ϵ by an odd multiple of π.

If we are agreed that the duplex vector is a feasible alternative to the vector with one direction only, two questions have to be answered. Firstly, is there any actual harm in the simpler idea? Secondly, are there any positive advantages in the more complicated one? I want to justify by examples the answering of both these questions by an emphatic affirmative.

In point of fact, the common plan leads continually to arguments that are logically indefensible. In many text-books we need read no farther than to the definition of the components x, y, z to find our teeth on edge: the statements that any vector can be resolved into components along the three axes, and that x, y, z are the amounts of these components measured in the directions of the axes, will not strike you as unfamiliar, and yet the second of them is obviously out of place in any book in which it has already been laid down that the amount of a vector is essentially positive. Again, in discussing the

collision of elastic spheres, do we not begin by assuming speeds of *approach* and speeds of *separation*, and do we not interpret without hesitation formulae in which some of these numbers are negative? I do not say that this procedure can not be justified even when vectors are by definition unidirectional, but I do say that in the majority of our books there is absolutely no pretence at justification, no hint of a realisation that definitions have been thrown overboard for the sake of a quick passage, and I do ask whether there is any simpler way of putting matters straight than by a frank recasting of the initial ideas.

In conclusion let me describe a development that is rendered possible only by the view of a vector that I am defending; I refer to the introduction of the notions of direction and angle into the space where coordinates and distances are complex numbers. There is no difficulty in defining vectors and their components, and in determining the projected product and the vector product of two vectors, and the projected square of a vector makes its appearance naturally: with rectangular axes the vector whose components are x, y, z has projected square $x^2+y^2+z^2$. What is it then that has a definite direction? Not a vector as such, for there is no satisfactory rule for distinguishing between the two square roots of an individual complex number. Not even a vector regarded as a multiple of a unit vector, and this for two reasons; in the first place $(7+4i, -2+i, -4+7i)$ is just as much the multiple of the unit vector $(2+3i, -1, -3+2i)$ by $2-i$ as of the unit vector $(-2-3i, 1, 3-2i)$ by $-2+i$, and if direction is to have any meaning these two unit vectors must surely have opposite directions; and in the second place, a vector for which $x^2+y^2+z^2$ is zero is not a multiple of any unit vector, and we should be unwilling to say that a nul vector has *no* direction. The concept to which direction is to be attached is that of a *measured vector*, that is, a vector associated specifically with one of the two square roots of its projected square. If r is an amount of a vector ρ, then kr is one amount of $k\rho$, and unless kr is zero, we can distinguish the measured vector $(k\rho, kr)$, the measured vector in which $k\rho$ is associated with kr, from the measured vector $(k\rho, -kr)$; the former of these is codirectional with the measured vector (ρ, r), but the latter is not. And by the direction of the proper measured vector (ρ, r) can be meant the class of all measured vectors codirectional with (ρ, r). The zero measured vector, that is, the measured vector (ρ, r) such that ρ has components $(0, 0, 0)$, has the one amount zero, and all directions, but every proper measured vector has a unique direction. If (ρ, r), (σ, s) are two measured vectors and p is one amount of the vector product ϖ of ρ and σ, then the angle ϵ is an angle from (ρ, r) to (σ, s) round (ϖ, p) if $rs \cos \epsilon$ is the projected product of ρ and σ and $rs \sin \epsilon$ is equal to p. In a plane with rectangular coordinates, the angles from (ρ, r) to (σ, s) are the values of ϵ satisfying simultaneously

$$rs \cos \epsilon = x_\rho x_\sigma + y_\rho y_\sigma, \quad rs \sin \epsilon = x_\rho y_\sigma - y_\rho x_\sigma.$$

Except for nul vectors the indeterminacy is the same in algebraic space, real or complex, as in real space. The peculiarity of a nul vector is not that it has no directions, but that it has only one direction; every vector in a nul direction is nul, but two nul vectors are in the same direction only if one of them is a multiple of the other. E. H. NEVILLE.

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39. *1.* I am now in my 92nd year (1872), still able to drive out for several hours; I am extremely deaf, and my memory of ordinary events, and especially of the names of people, is failing, but not for mathematical and scientific subjects. I am still able to read books on the higher algebra for four or five hours in the morning, and even to solve the problems. Sometimes I find them difficult, but my old obstinacy remains, for if I do not succeed to-day, I attack them again on the morrow. . . .—M. S. *Loc. cit.*, p. 364.

[The last words of her autobiography are: "I am perfectly happy."]

Miss H. M. Cook opened a discussion on:

THE POSITION OF COMMON LOGARITHMS IN MATHEMATICAL TRAINING.

In discussing the teaching of other branches of their science, mathematicians often assume that children of school age are capable of using tables of common logarithms with considerable accuracy. This is by no means always the case. If the discrepancy is due to essential differences in the development of children in different parts of the country any attempt to make practice uniform must of course meet with failure, but I would claim that some appreciation of the use of logarithms is possible for every child in our schools.

It remains to decide at what stage relatively to other mathematical subjects this particular study should be introduced, and three main questions must influence our discussion:

(1) What is the present position of common logarithms?

(2) What ought the position to be?

(3) Ought the Mathematical Association to take active steps to hasten the movement from the present position to the ideal position?

The present position, at any rate in the smaller schools, seems to depend on the particular external examination for which pupils are encouraged to work. The Oxford and Cambridge Local Examinations now include questions on the use of tables of common logarithms, without requiring formal pıoofs of the Index Laws, and so do the papers set for the Northern Universities Joint Matriculation. This corresponds to the recommendations made by the Mathematical Association Committee, but the London University—proverbially slow to alter its regulations—makes no mention of logarithms (except to forbid their use) in the syllabus for the compulsory Matriculation papers. Nor are they compulsory for Responsions. There are still a few students entering the Mathematical departments of our University Colleges who know practically nothing of the use of tables, and I think it is fair to infer that their class-mates who enter other faculties, or begin business life on leaving school, have received no fuller mathematical teaching as a part of their general education, and will in most cases never have any idea as to how logarithms simplify calculation.

In the schools in which the use of logarithm tables is taught, there is still considerable difference of opinion and practice as to its proper place in the curriculum. I have known boys with no particular mathematical bent—indeed very backward in all forms of book-work—who greeted the subject as a pleasant change from commercial arithmetic, and attained considerable mechanical skill in using tables after a very slight numerical introduction to fractional indices, some time before their algebraical powers had advanced far enough for them to appreciate the excellent chapter in Messrs. Godfrey and Siddons book, which gives a graphical introduction to the subject.

Personally, I like to use this graphic method. In any case the demands of science teachers suggest that logarithms should come early in the algebra course—see the General Mathematical Syllabus for non-specialists, Part I. Certainly geometrical progressions are very artificial if taken before logarithms, as either the number of terms or the ratio must be limited in any example set for children to work out.

The use of some form of tables ought to be taught in every school. Children may themselves construct tables of squares—a much more stimulating exercise than haphazard practice in simple multiplication, and they may then be taught to appreciate tables of square roots. Logarithm tables naturally follow, and if all the business community understood their tables of Compound Interest, Annuities etc., would be less mechanically and more intelligently

used. This is the "outlook" justification for logarithms, as a school subject, and suggests that they could be profitably taught in the new Continuation Schools. I have myself talked with children, who, having passed Standard VII. and been admitted as factory hands, were required by their employer to attend evening classes. The teacher was the headmistress of the elementary school, and their complaint was that they learned no new arithmetic, but simply worked through the Standard VII. sums again. To say the least of it, this seems to have been an opportunity lost.

Ought we to approach the London University and other educational authorities to make the teaching of the use of common logarithms more general?

H. M. Cook.

A discussion ensued, in the course of which the following comments were made:

Prof. T. P. Nunn—The exclusion of logarithms from the syllabus of the London Matriculation Examination in mathematics is undoubtedly a serious and mischievous thing. The explanation appears to be that the University Board of Studies in Mathematics assume that the doctrine of logarithms necessarily rests on the doctrine of fractional indices; since the latter is not required, the former cannot be included. Miss Cook quite properly rejects the excuse, but she weakens her position, I fear, by accepting the assumption that gives it what force it possesses. That assumption is, of course, entirely untrue—as is proved by the historical fact that Napier had worked out his theory of logarithms many years before Wallis invented, in a totally different connection, the fractional index notation. In fact, given the ideas (1) of a doubly endless geometric sequence with unity as its middle term and (2) of a doubly endless arithmetic sequence with zero as its middle term, it is easy to work out an entirely satisfactory theory of the logarithmic tables—and that is, surely, all that even the purist can demand before permitting us to place the tables in our pupils' hands.

Prof. Milne commented on the necessity of a knowledge of trigonometry and logarithms for students entering a university, it not being at present a compulsory subject in the papers on Mathematics in various Higher Certificate and Matriculation Examinations.

He also referred to the unexpected applications of mathematics in industry, instancing the fact that "scales of notation" are fundamental in the mathematical basis of the textile industry.

Miss E. M. Read had always taught logarithms before G.P. On showing a series such as 2, 4, 8, 16 . . . to 50 terms, she found that the attempts at guessing the 50th term were very wide of the mark. She then worked out

$$2^{50}=(10^{0\cdot3010})^{50}=10^{15\cdot05}=10^{15}\times1\cdot122.$$

To drive home the magnitude of this result she took 1122000000000000, which did not, however, seem so convincing to her pupils as 1122386294716218, where the last twelve digits were written down at random.

Mr. M. Finn—Boys of 13 can be led in ten minutes to *discover* for themselves the desirability of the use of the zero index, negative indices and fractional indices, after they have grasped such statements as $a^2\times a^3=a^5$, $a^7\div a^3=a^4$ and $(a^2)^3=a^6$. Write in a vertical column the series 1,000,000, 100,000, 10,000, 1,000, . . . 1/1,000,000. In a parallel column, let the first few terms of the series be replaced by their equivalents, 10^6, 10^5, 10^4, ... Zero and negative indices will immediately be suggested by the boys in order to complete the series in this *shortened* notation.

Now take the series 10^8, 10^4, 10^2, 10^1, ... In order to continue it, we are compelled to introduce $10^{1/2}$, $10^{1/4}$, etc. We are thus led to fractional indices and their meaning.

Tests should now be applied to see if the rules which apply to positive integral indices also hold good for these " newcomers."

No further real advantage of any sort is obtained by the boys from a formal study of the " Theory of Indices."

The idea of logarithms and the rules concerning the log. of a product, etc., should be introduced by merely paraphrasing the corresponding statements in the Theory of Indices.

In her reply, Miss COOK welcomed Prof. Milne's suggestion of an attack on London University from the flank. She recognised that London was not the only offender, but in view of its external degrees it had a wider influence, and might therefore be considered the worst offender.

She would not necessarily postpone all mention of the geometric sequence till after logarithms, but she thought the formula for the sum of the series could be usefully employed only when logarithms were known and used.

No exhaustive treatment of the Theory of Indices need precede the use of logarithmic tables—indeed, if taught as part of an arithmetic course, numerical bases might be used throughout, and this would probably be done in elementary and continuation schools, but the fundamental index law must be explained.

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40. *J.* [Arago's] character was noble, generous, and singularly energetic; his manners lively and even gay. He was a man of very general information, and, from his excitable temperament, he entered as ardently into the politics and passing events of the time as into science, in which few had more extensive knowledge. On this account I thought his conversation more brilliant than that of any of the French savans with whom I was acquainted. . . . The Marquis de la Place was not tall, but thin, upright, and rather formal. He was distinguished in his manners, and I thought there was a little of the courtier in them, perhaps from having been so much at the court of the Emperor Napoleon, who had the highest regard for him. Though incomparably superior to Arago in mathematics and astronomical science, he was inferior to him in general acquirements, so that his conversation was less varied and popular. . . . M. Arago had told M. de la Place that I had read the *Mécanique Céleste,* so we had a great deal of conversation about astronomy and the calculus, and he gave me a copy of his *Système du Monde,* with his inscription, which pleased me exceedingly. . . .—M. S. *Loc. cit.*, pp. 108-109.

[Some may think that the Preface to Laplace's *Théorie Analytique des Probabilités* borders on the sycophantic; at any rate it seems to bear out Mrs. Somerville's suggestion as to there being "a little of the courtier" about him.

A

NAPOLÉON-LE-GRAND

Sire,—La bienveillance avec laquelle VOTRE MAJESTÉ a daigné accueillir l'hommage de mon Traité de *Mécanique Céleste,* m'a inspiré le desir de LUI dédier cet Ouvrage sur le Calcul des Probabilités. Ce calcul délicat s'étend aux questions les plus importantes de la vie, qui ne sont en effet, pour la plupart, que des problèmes de probabilité. Il doit, sous ce rapport, intéresser VOTRE MAJESTÉ dont le génie sait si bien apprécier et si dignement encourager tout ce qui peut contribuer au progrès des lumières, et de la prospérité publique. J'ose La supplier d'agréer ce nouvel hommage dicté par la plus vive reconnaissance, et par les sentiments profonds d'admiration et de respect, avec lesquels je suis,

Sire,

De Votre Majesté,

Le très-humble et très-obéissant
serviteur et fidèle sujet,

LAPLACE.]

THE TEACHING OF MECHANICS TO BEGINNERS.

BY R. C. FAWDRY.

I AM much honoured by the invitation of your Council to open a discussion on the teaching of Mechanics to beginners, and as I am probably much more interested in hearing what you have to say on the subject than you are likely to be in listening to me, I hope many of you will take part in the discussion, and therefore I propose to be decidedly brief.

There has been much change in the teaching of Mathematics during the last twenty years, in some cases not altogether for the better; but in the case of Mechanics I venture to think that the changes have been most beneficial, and that the efforts to breathe life into dead bones have been most successful.

I say nothing to-day on the claims of Mechanics to form a part of any liberal education: that I think is generally conceded, but it is no less a fact that even now many boys leave school without any knowledge of the subject at all.

I propose to base my remarks on four axioms:

(1) That our treatment shall be based upon experiment.

(2) That we should build on the knowledge our pupils have already acquired.

(3) That we should as far as possible follow the historical development of the subject.

(4) That we should illustrate by simple examples dealing with familiar objects, and avoid examples which are mainly exercises in Algebraic or Trigonometric manipulation.

Shall we begin with Statics or Dynamics?

If we accept my first axiom I think we are bound to begin with Statics. Experiments are numerous, they require but simple apparatus, and they are easily performed. In Dynamics the apparatus is more complicated, the number of possible experiments is limited, and they are difficult to perform with accuracy.

Until quite recently Mechanics was often taught without any apparatus at all, and even now many teachers fight shy of the experimental work. I remember years ago that I passed the London Matriculation Examination in Mechanics without ever having done an experiment, and in Chemistry without ever having handled a test tube.

If a mathematical laboratory is available so much the better; if not, many statical experiments can be done in a class-room, and if that is impossible the master can do the experiments before the class, read out his observations, and let the boys write an account of the experiment, do the calculations, and deduce the results.

A few half-metre measures, some weights, a few retort stands, some dynamometers, and we have nearly all the apparatus necessary. A convenient dynamometer which reads up to 400 grs. is the Rintoul Dynamometer. Suppose then we start with Statics. We begin with some introductory practical work: the use of the dynamometer for measuring forces, the meaning of a tension, the idea of rigidity, centre of gravity and pressure.

What comes next? For generations all text-books started with the parallelogram of forces—they must have a proof, so they had to begin with the parallelogram of velocities or they fell back upon a horrible nightmare called Duchayla's proof, which was enough to make any self-respecting boy abandon the subject in despair. Years ago Canon Wilson, who was ahead of his generation in the teaching of Mechanics as well as Geometry, advocated that the subject should begin with Moments, and if we accept my second axiom this is undoubtedly right, for every boy is familiar with a see-saw if not with a balance. This also is in accordance with historical order: the principle of the lever was known generations before the parallelogram of forces, and we can can interest our pupils by telling them of the ingenious proofs devised

by Archimedes and afterwards by Stevinus, while allowing them to obtain the results by their own simple experiment with a half-metre scale.

Here we introduce quite simply the meaning of the words Resultant and Equilibrant, and omit that fatuous division of levers into three classes, which Lord Kelvin had learnt, "But which of them is which," said he, "I cannot for the life of me tell you."

Now we have the material for endless examples, and here we lay the foundations of success by insisting that every solution shall contain (1) a statement of the particular body whose equilibrium is being considered; (2) a list of the forces acting on that body; (3) a statement of the principle that is being employed in the solution.

After dealing with moments in which the forces act at right angles to the arm, we pivot a circular piece of stiff cardboard at its centre, balance two equal weights, then move one to act at a point vertically below. Will the cardboard turn? Need the text-book give the show away by stating the answer? If so, it is the private student who is responsible, and he has been responsible for spoiling many a text-book. If he cannot do the experiment himself, surely he can find some one who can tell him.

According to my second axiom, Machines should come next. All boys know something about them, and all are interested in them. In my school days they came at the end of the book, and so I thought they must be very difficult; and I always felt there must be something about them I couldn't understand, because they seemed so easy—but then they were such perfect machines, always weightless and always frictionless.

We begin by explaining to our pupils what Mathematicians mean by work, and then we are able to discuss the efficiency of all sorts of machines: levers, pulleys, inclined planes, winches, screwjacks, and so forth. Again we omit the division of pulleys into three classes, and sacrifice without a sigh the wonderful third system by which we could raise infinite loads with infinitesimal effort, and leave the famous Cambridge lecturer no longer any pretext for setting up his apparatus with a concealed nail in order to make it keep steady. Instead of this we introduce the Weston differential pulley, which really works and is almost equally wonderful.

We can now either complete our course on Moments by taking C. of G., or we can tackle the triangle of forces. Here we introduce the notion of a vector, and get an inkling of the method by which it may be possible to find the resultant of two forces not parallel by considering the analogous case of two vectors representing a country walk.

Here too we can add to the interest of the lesson by saying something about the various efforts that were made before Newton's time to solve this problem, and it would be a great pity to omit the ingenious argument of Stevinus, with his endless chain passing round an inclined plane.

The way is now open to discuss the resolution of forces and the polygon of forces, and a final chapter on the elementary parts of friction completes our elementary course of Statics.

Now we come to Dynamics.

Here we enter more controversial ground, and as it seems impossible for the various schools of thought to agree, we must perforce agree to differ.

We begin with velocity. What more natural than to finish Chapter I. by discussing relative velocity. So thought the previous generation, but obviously they realised that it was useless at that stage to get pupils to understand it, so they gave it a definition and set examples on it. Now that we know that only Einstein can understand it we put it at the end instead of at the beginning. Yet only last year a well-known examination had a question on relative velocity as the first question in an Elementary Dynamics paper.

One of the most important advantages resulting from recent changes in mathematical teaching is the introduction of the elements of the Calculus into the school work, and when we teach Dynamics we can do much to help

the Calculus. We remember that our pupils later on will be able to use the Calculus in their Dynamics, and we can teach them the meaning of the Calculus by freely using graphical methods of differentiating and integrating in our treatment of velocity.

The Fletcher trolley is invaluable for our experimental work, but as it is doubtless familiar to you I need only mention it. We soon reach the stage of drawing graphs showing the relation between distance and time, first with velocity constant and then variable. From these graphs we find the velocity from the gradients of the tangent, and having shown that distance can be estimated from a velocity-time graph by finding an area, we arrive at the formula $s=\frac{u+v}{2}.t$ for uniform acceleration, and at once do our best by discussing cases in which the acceleration is not constant to drive out the idea that this formula is of universal application. No reference should be made to any other formula for uniformly accelerated motion until a second reading of the subject or most of the work becomes at once mere mechan cal substitution of figures for letters.

Falling bodies naturally come next, and here we can adhere very closely to Galileo's methods—his ingenious devices for diluting gravity, for measuring time, and his skilful use of the graphical method are all full of interest.

At this point we have to break our historical order, for it is obviously undesirable at this stage to discuss motion in a circle, the investigation of which was due to the genius of Huygens, to some extent a contemporary of Galileo.

There are some who advocate that we should now go on to the Energy principle, on the grounds of its importance and on the assumption that it was merely an accident that it was not discovered before the Laws of Motion. Personally I cannot feel that it was an accident, and I know of no satisfactory way of treating it without previously discussing the Laws of Motion.

If we take next the first Law of Motion, we can begin again with Galileo, who, as Mach points out, really discovered it when he discussed the behaviour of a body falling down an inclined plane and then ascending another inclined plane, and yet reaching the same height, however small the inclination of this plane is made.

The first law deserves much more attention than is generally given to it, and many examples may be given of bodies moving under the action of forces which have no resultant.

We can still follow Galileo by now discussing the motion of a body projected in some direction other than the vertical, and indeed projectiles are probably treated better by Galileo's method than by resolving the initial velocity into horizontal and vertical components.

The second law is not easy to deal with experimentally, but certainly Fletcher's trolley is the best apparatus for our purpose. The board is tilted until a body W slides down with uniform speed when fastened by a string to a weight (w) over a pulley at the top. The forces on W are $W \sin \alpha - F - T = 0$ and $T = w$; $\therefore W \sin \alpha - F = w$. When w is detached, the force taking the body down the plane is $W \sin \alpha - F$, and this equals our w. From the graph drawn by the vibrating bar we deduce the acceleration, and find that it is proportional to the force producing it. Our law is then found in the form $\frac{P}{W}=\frac{a}{g}$, and we use this instead of Newton's Law.

What are we to do about Inertia and Mass? In my opinion it is a waste of time to introduce the question of measuring mass at this stage—we can talk about it, give our pupils some idea of its meaning, explain that it is not the same as weight, and there leave it until he is much more capable of understanding it. So too we need have nothing to do with the poundal.

The arguments on these matters have been given with so much force by

Professor Perry in the report of the discussion at the British Association meeting at Johannesburg in 1905 (published by Macmillan) that nothing I can say will strengthen the case. At that meeting Professor Boys said that the poundal was the most terrible trap ever set for the unwary student—so it is—and after trying to teach poundals for about ten years I gave it up with most satisfactory results—my temper improved, my pupils revived, and their progress in Mechanics doubled in pace.

I think, however, that it is desirable to reduce to a minimum the time spent on the second and third Laws of Motion, so that we can devote our savings to the acquisition of facility in dealing with problems by the application of the Energy principle. If our pupils get into the habit of calculating the acceleration of a moving body, it is difficult to get them out of it. Even my generalisation, well known to them for its inaccuracy, that everything in pure mathematics is proved by Projection, and everything in Dynamics by Energy, does not always have the desired effect.

Here again we must insist that our pupils always state definitely the forces acting on the particular body whose motion is being discussed, and insert them in a diagram before they write down their equations. From that point of view examples on the second and third laws are valuable, and should not altogether be neglected. Of course our deduction of the Energy equation from $\frac{P}{W}=\frac{a}{g}$ leads us to the expression $W\frac{v^2}{2g}$ for K.E., instead of $M\frac{v^2}{2}$, with the inestimable advantage of doing away with that awful conundrum—must I multiply or divide my answer by 32 to get foot-lbs.?

I can only briefly allude to Momentum, our formula for which will be $W\frac{v}{g}$, but I should like to recommend the Ballistic pendulum as a suitable experiment. The idea of velocity as a vector will then be introduced, followed by the parallelogram of velocities, projectiles and relative velocity.

Before giving any geometrical construction for finding relative velocity, it is most desirable to give our pupils a clear idea of its meaning by plotting out the relative positions of two moving bodies.

Were it not for the demands of examining bodies I should be inclined to omit motion in a circle in an elementary course, but if we teach it to beginners it requires very careful handling, especially in the use of the term centrifugal force; if we use it at all we must avoid giving the impression that the body is acted upon by a force causing it to fly outwards; we want to impress upon the student that he must look for a force inwards to make the body take the curve, and confine the term centrifugal force to the outward reaction on the constraint.

R. C. Fawdry.

In the subsequent discussion, Dr. Brodetsky remarked—I agree in the main with the speaker's scheme of work, and in fact have imitated his treatment as contained in his books. I think that the question of Statics before Dynamics, or *vice versa*, depends on type of course. If experimental, Statics first; if theoretical, Dynamics first. I disagree with his views on Relative velocity. This is now commonly discussed in connection with aviation, for example, and people realise its effects in, for instance, air raids (when Zeppelins get lost because of wind) or cross-Atlantic flights. I think that this example should be easy at first, yet later it is an advantage to have examples that need considerable labour.

Mr. Fawdry summed up as follows:—The fact that we have had so little criticism in the discussion is clear evidence of the change that has taken place in the teaching of Mechanics. I am much disappointed, for I quite expected to be rent asunder this afternoon, and when I think of what might have been said, for instance, about Inertia, Mass and the Poundal, I am astonished at your moderation. I think, indeed, that at these meetings we have too little

opposition, and our discussions would be more interesting if those who disagreed would be more violent in their attacks.

There seems to be little need for us to impress upon our students the divergence between the results we obtain by experiment and by calculation. Their own experience is sufficiently instructive on that point.

I did not advocate omitting Relative Velocity even in a first course; I suggested that it comes better at the end than in the first chapter. Boys find Relative Velocity difficult to understand properly. It is easy to give geometrical definition and for them to do examples, but that is a different thing from understanding what they are doing. Let them begin at any rate by considering motion relative to the earth as their base, taken as fixed for that purpose.

I deprecate the use of the Calculus for beginners in Dynamics. If we treat the subject graphically we are teaching our pupils the meaning of their Calculus. If we use the Calculus too soon the work is likely to become mere manipulation of symbols.

I agree that this course leaves Rotational Dynamics to a later period, but nothing like so late as it used to come. The growing use of the Calculus enables Rotation to be taught in a school course, and I think that it is desirable even in a first course to assume that the energy principle holds good for such questions as those dealing with rotating fly-wheels.

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41. *K.* [Dr. Wollaston] was remarkably acute in his observations on objects as we passed them. "Look at that ash tree; did you ever notice that the branches of the ash tree are curves of double curvature?"—M. S. *Loc. cit.* p. 148.

42. "We discussed together the various illustrations that might be introduced: I suggested several, but the selection was entirely her own. So also was the algebraic working out of the different problems, except, indeed, that relating to the numbers of Bernouilli, which I had offered to do, to save Lady Lovelace the trouble. This she sent back to me for an amendment, having detected a grave mistake which I had made in the process.

"The notes of the Countess of Lovelace extend to about three times the length of the original memoir.* Their author has entered fully into almost all the very difficult and abstract questions connected with the subject."—Babbage, *Passages from the Life of a Philosopher*, 1864, p. 136.

43. *L.* La Place had a profound veneration for Newton; he sent me a copy of his *Système du Monde*, and a letter, dated 15th August, 1824, in which he says:

"Je publie successivement les divers livres du cinquième livre qui doit terminer mon traité de *Mécanique Céleste*, et dans cela je donne l'analyse historique des recherches des géomètres sur cette matière, cela m'a fait relire avec une attention particulière l'ouvrage si incomparable des principes mathématiques de la philosophie naturelle de Newton, qui contient le germe de toutes ses recherches. Plus j'ai étudié cet ouvrage plus il m'a paru admirable, en me transportant surtout à l'époque où il a été publié. Mais en même tems que je sens l'élégance de la méthode synthétique suivant laquelle Newton a presenté ses découvertes, j'ai reconnu l'indispensable nécessité de l'analyse pour approfondir les questions très difficiles que Newton n'a pu qu'effleurer par la synthèse. Je vois avec un grand plaisir vos mathématiciens se livrer maintenant à l'analyse et je ne doute point qu'en suivant cette méthode avec la sagacité propre à votre nation ils ne seront conduits à d'importantes découvertes."—M. S. *Loc. cit.*, p.181.

*[Ada Augusta, Byron's only child, translated General Menabrea's Memoir on the Analytical Engine. She married Lord King, afterwards Earl of Lovelace.]

THE GRAPHICAL TREATMENT OF DIFFERENTIAL EQUATIONS.

BY DR. S. BRODETSKY.

(*Continued from p.* 8, *January*, 1920.)

(v) But it must not be supposed that a straight line solution is always an asymptote. Such is the case in Figs. 7 and 8, as indicated clearly by the information given by 7 (*a*), 8 (*a*). In the case of $y_1 = xy$, however, although $y=0$ gives $y_1 = 0$, $y_2 = 0$, and is therefore a straight line solution, Fig. 9 (*a*) indicates that it is not an asymptote, and the curves are as in 9 (*b*), as can be easily verified analytically.

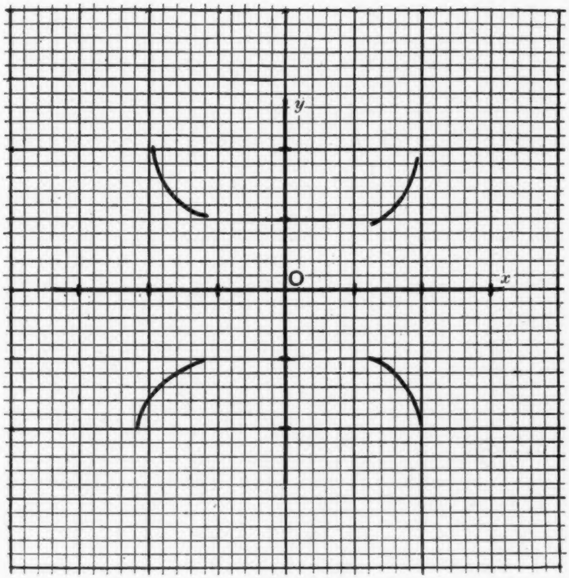

FIG. 9 (*a*).

(vi) A more complicated case is

$$y_1 = x + y^2,$$

which is again insoluble by any of the standard forms. We get

$$y_2 = 1 + 2yy_1 = 1 + 2y(y^2 + x).$$

Hence $y_1 = 0$ along the curve $x = -y^2$, dotted in Fig. 10 (*a*). $y_2 = 0$ along the curve $x = -y^2 - \dfrac{1}{2y}$, dashed in 10 (*a*). The inflexion locus is thus asymptotic to the locus $y_1 = 0$. We get the general information shown in 10 (*a*). The curves are given in 10 (*b*). There is a limiting curve asymptotic to the loci $y_1 = 0$, $y_2 = 0$. below the axis and lying between them.

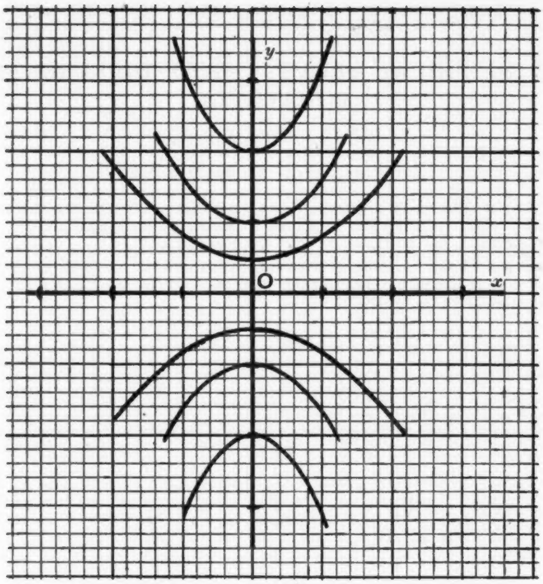

FIG. 9 (b).

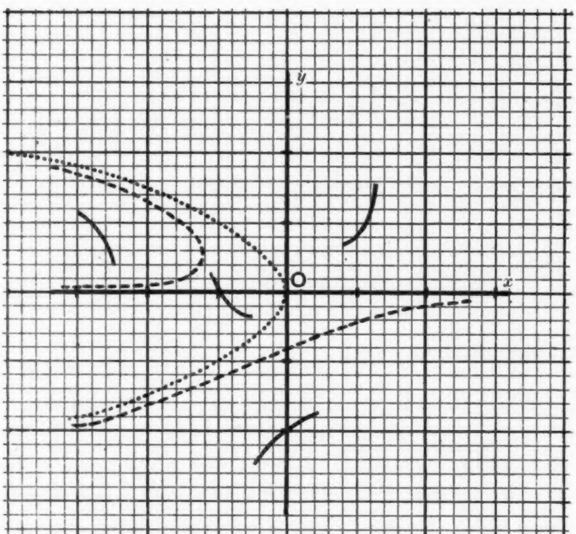

FIG. 10 (a).

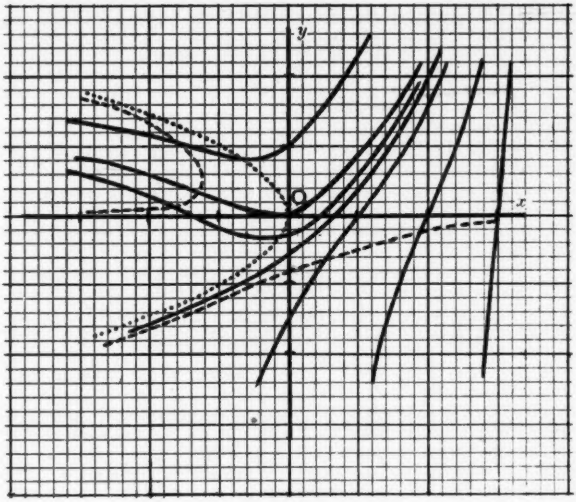

FIG. 10 (b).

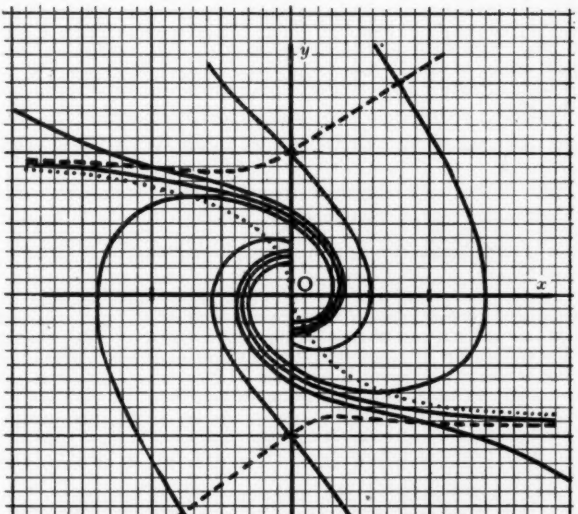

FIG. 11.

(vii) The differential equation mentioned at the beginning of this paper, viz.

$$\frac{dr}{dx} = -y, \quad \text{or} \quad \frac{dy}{dx} = -\frac{x}{y} - (x^2+y^2)^{\frac{1}{2}},$$

gives $y_1 = 0$ along the curve $r = -\cot\theta$, where $x = r\cos\theta$, $y = r\sin\theta$, and $y_2 = 0$ along the curve $y^4 - xyr - r^2 = 0$.

The compartments and general forms are given in Fig. 4. The curves are sketched in Fig. 11. These curves were actually used to obtain a graphical solution of the dynamical problem giving rise to this equation, and the results were found to conform to the motion observed when a lamina, such as a postcard, falls in two dimensions in air.

A few exercises are suggested for practice by means of the method sketched thus far.

(viii) $y_1 = y + e^x$ (Soluble analytically);

(ix) $y_1 = x - y^3$ (Insoluble analytically);

(x) $y_1 = x^2 - y^3$ (Insoluble analytically);

(xi) $y_1 = y^2(x+y)$ (Insoluble analytically);

(xii) $\dfrac{dr}{dx} = -\dfrac{ay}{(1-x^2)^{\frac{1}{2}}}$ (an extension of the equation (vii) in § 5, and required for the solution of a more extended problem in resisted motion).

6. We have restricted ourselves to the type $y_1 = f(x, y)$, where f is a one-valued function of the coordinates (x, y), or where, as in (vii), § 5, a possible ambiguity is avoided by the use of a definite interpretation. When the differential equation in its general form $F(y_1, x, y) = 0$ admits of more than one value of y_1 at any given point (x, y), two alternatives present themselves. On the one hand, the different values may belong to analytically different families of curves, or the given equation is reducible. Thus, if we have $y_1^2 - x^2 = 0$, we get $y_1 = +x$ and $y_1 = -x$. These are two distinct equations which really have little relation to one another. But on the other hand, the different values of y_1 may depend on the same analytical form, the multiple values arising from an ambiguity, as *e.g.* in the equation $(a^2 - x^2)y_1^2 - x^2 = 0$. Here we have

$$y_1 = +x/(a^2-x^2)^{\frac{1}{2}} \quad \text{and} \quad y_1 = -x/(a^2-x^2)^{\frac{1}{2}},$$

and the difference in sign can be assigned to the occurrence of the square root $(a^2-x^2)^{\frac{1}{2}}$.

In order to produce a consistent method we shall not follow the common practice of saying that an equation like $y_1 = x/(a^2-x^2)^{\frac{1}{2}}$ or $y_1^2 - xy_1 + 1 = 0$, or any other such equation defines a family of curves. This is clearly incorrect, and for the geometrical method of this paper is in addition very embarrassing. We shall define a *family of curves* to be such *an infinite set of curves as do not intersect one another*, so that at any point (x, y) there is one definite direction of the tangent (unless no such curve exists at all at the particular point in question, as we shall see in some of the illustrations). Basing ourselves on this idea of families of non-intersecting or non-interfering curves, we shall therefore say that a differential equation of the first order defines as many families as the degree of the highest power of y_1 in the equation when rationalised. In some cases the families merely coexist on the same plane. But when the equation is not reducible to rational forms of lower degrees, then the families may have interesting relations, giving rise to singular solutions, envelope loci, cusp loci, etc.

The slight modification we have introduced in the definition of a family of curves is obviously useful from the practical point of view. For in practice the particular problem giving rise to the differential equation in question also defines the particular signs to be given to any ambiguities in the equation, and so the solution is required for the definite value of y_1 thus determined.

(*To be continued.*)

CO-ORDINATE GEOMETRY IN SCHOOLS.

BY W. J. DOBBS.

(*Continued from p.* 388, *October*, 1919.)

§ 7. **(r, x) Equations.** It will be said that I have omitted all reference to the focus, and that if the student is to make any study of mathematical astronomy he will require the focus properties of the ellipse. Yes; but he can get on very well without the directrix.

When Kepler was seeking for an explanation of the varying distance between the Earth and the Sun, he examined among other possibilities an assumed circular path of the Earth round the Sun excentrically situated. In the case of any curve which is symmetrical with respect to the x axis, it seems reasonable to express the equation of the curve in the form of a relation connecting r with x. The circle of radius a and centre $(c, 0)$ has for its equation

$$x^2+y^2=2cx+a^2-c^2,$$

$$\text{i.e.}\quad r^2=2cx+a^2-c^2.$$

Thus the growth in r^2 is proportional to the growth in x. If we take an conic and suitably choose our origin S situated in an axis of symmetry, we can obtain a much more simple equation connecting r with x. We will take as the equation of our conic

$$y^2=Bx+Cx^2. \qquad \text{(vi)}$$

Transferring to $(h, 0)$ as origin without change of x axis, the equation of the conic becomes

$$y^2=B(x+h)+C(x+h)^2,$$

$$\text{i.e.}\quad y^2=Bh+Ch^2+(B+2Ch)x+Cx^2,$$

$$\text{i.e.}\quad r^2=Bh+Ch^2+(B+2Ch)x+(C+1)x^2.$$

Now let us examine whether we can choose h so that the expression on the right may be a perfect square. The condition is

$$(B+2Ch)^2=4(Bh+Ch^2)(C+1),$$

$$\text{i.e.}\quad 4Ch^2+4Bh-B^2=0.$$

If the conic is a parabola, *i.e.* if $C=0$, there is but one possible value of h, namely $\frac{1}{4}B$. In this case the equation of the conic reduces to

$$r^2=(\tfrac{1}{2}B+x)^2.$$

If the conic is a central conic, *i.e.* if C is not zero, we have a quadratic to determine h, and the condition for real values of h is

$$4B^2(1+C) \text{ positive,}$$

$$\text{i.e.}\quad C+1 \text{ positive.}$$

This condition merely excludes the minor axis of an ellipse.

Half the sum of the two values of h is $-B/2C$, showing that the two foci are situated in the x axis at equal distances on opposite sides of the centre. When $C=-1$, *i.e.* when the conic is a circle, the two foci coincide at the centre of the circle.

The quadratic for h shows that

$$Bh+Ch^2=\tfrac{1}{4}B^2$$

and

$$B+2Ch=\pm B\sqrt{1+C}.$$

Thus the equation of the conic becomes

$$r^2 = (\tfrac{1}{2}B \pm \sqrt{C+1} \,.\, x)^2;$$

$$\therefore \; r = \pm l \pm ex,$$

where $l = \tfrac{1}{2}B$ and $e = \sqrt{C+1}$.

A little consideration shows that, admitting negative values of r, $\pm l$ may be taken as positive without loss of generality, and that the sign before ex may also be taken as positive, since reversing the x axis would change $l - ex$ into $l + ex$.

We have now the equation of the conic in the very simple form

$$r = l + ex.$$

For the parabola $C = 0$ and $e = 1$; for an ellipse C lies between 0 and -1, and e is less than 1; for a hyperbola C is positive and e greater than 1.

If we take SO along the x axis equal to $-l/e$ and measure X from O, continuing to measure r from S, the above equation takes the form

$$r = e \,.\, X.$$

If $e = 1$, the vertex A is the middle point of SO. Otherwise, the vertices A and A' divide SO internally and externally in the ratio e to 1. The circle on AA' as diameter is the auxiliary circle, the equation of which may be expressed in the form

$$r = e \,.\, R,$$

where r is measured from S and R from O.

From any point T on the tangent at P draw TM perpendicular upon SP and TH perpendicular upon the x axis. Also draw PK perpendicular upon

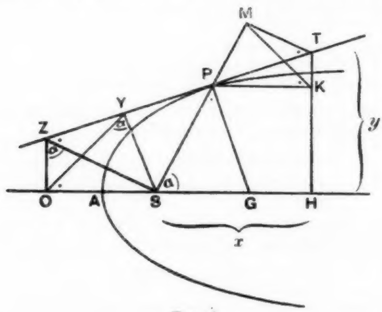

FIG. 4.

TH. The geometrical interpretation of the statement $\dfrac{dr}{dx} = e$ is

$$PM = e \,.\, PK,$$

whence $SM = e \,.\, OH = l + ex = eX$, where x and X have reference to the point T (not P). This is Adams' property of the tangent to a conic, and it gives at once the equation of the tangent. For, projecting SH and HT upon SP, we have

$$l + ex = x \cos \alpha + y \sin \alpha,$$

or, in radial coordinates,

$$l + er \cos \theta = r \cos (\theta - \alpha).$$

Draw PG the normal at P meeting the x axis at G. Then PG is the tangent at P to the circle on PT as diameter, and it is easily seen that the triangles SPG and PKM are similar.

$$\therefore \; SG = e \,.\, SP = er,$$

and the projection of SG upon SP is therefore ex, where x and r have reference to the point P. As $SP = l + ex$, the projection of PG upon SP is constant, being equal to l.

Draw SZ perpendicular to SP to meet the tangent TP at Z. Then, when T is taken at Z, SM becomes zero.

$$\therefore \; \text{the } x \text{ of } Z = -\frac{l}{e}.$$

Thus the locus of Z is the fixed straight line $x = -l/e$ (the directrix) meeting the x axis at O.

Draw SY perpendicular upon the tangent ZP. Then SP is the tangent at S to the circle on ZS as diameter, and it is easily seen that the triangles SPG and YOS are similar.

$$\therefore \; SY = e \,.\, OY.$$

Thus, when $e = 1$, the locus of Y is the perpendicular bisector of OS, and when e is not unity the locus of Y is the auxiliary circle.

A few general remarks on (r, x) equations may not be inappropriate. Consider the two related curves

$$\left\{\begin{array}{ll} & y = f(x) \\ \text{and} & r = f(x) \end{array}\right\}.$$

Only points on the former, for which the ordinate is numerically not less than x, correspond to real points on the latter; *i.e.* points on the former

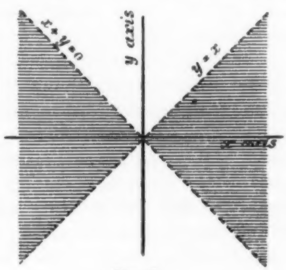

FIG. 5.

situated within the shaded region have no real corresponding points on the latter.

Again, points other than the origin common to the former curve and the y axis are points of contact of the two curves. Also, points common to the former curve and the lines $y = \pm x$ correspond to points common to the latter curve and the x axis.

The application of the above to the tangent $y = l + ex$ and the conic $r = l + ex$ are obvious. Notice that in the parabola and ellipse r is positive throughout; in the hyperbola, r is positive for one branch and negative for the other.

The intercept on the x axis between the origin and the normal for the curve $r = f(x)$ is given by $r\dfrac{dr}{dx}$, and has the same value as the subnormal for the curve $y = f(x)$. For instance, a parabola having its axis of symmetry along the x axis corresponds to a circle. Again, the subnormal for the line $y = l + ex$ is ey; this gives $SG = e \,.\, SP$ for the conic $r = l + ex$.

W. J. DOBBS.

Obituary.

G. W. Palmer.

By the untimely death of G. W. Palmer, Master of the Royal Mathematical School, Christ's Hospital, the teaching world has suffered a heavy loss. His chief interest lay in academic mathematics, but by force of circumstances his earlier post as Head of the Military Side at Clifton College compelled him to turn his attention to the practical rather than to the formal side of the subject. This exercised an overmastering effect upon his mental vision. His outlook was gradually turned into a different channel by his practical every-day teaching; and his remarkable pioneering work on Arithmetic, with his notable papers on the teaching of that subject, give ample evidence of originality. His contributions to the Teaching Committee were characterised by freshness, pungency, and non-compromising denunciation, where questions of inaccuracy and over-stepping the limits of the data were concerned. We have heard it said that he well deserved to be called the "Father of Arithmetic" in English education.

However that may be, although the removal to Christ's Hospital brought him back into direct touch with the more academic forms of mathematical work, he instituted and maintained a high standard of mental and applicational work in the Arithmetic of the school, and in that subject the master-hand was seen at every turn. By the stress we have laid on his contributions to the literature of one branch of elementary Mathematics, we do not wish to imply that his interest in his work was thereby limited. It may be said, indeed, that he inaugurated a new era in the teaching of Mathematics at Christ's Hospital. More time was given to important principles and fresh ideas, elaborate development in any one direction was carefully avoided, and with the happiest effect his teaching was leavened by his practical acquaintance with many subjects, and in particular by his wide knowledge of scientific geography. The natural outcome was an increased interest throughout the school, the attainment of a high general level, and a considerable advance in individual achievement at the universities and elsewhere.

His colleagues will always remember Palmer as a man of keen intellect, an enthusiast with high ideals, incapable of harbouring an unkind thought, characteristically generous in disposition, exquisitely sensitive to beauty, and gifted with a fine sense of humour.

He was singularly fortunate, more so than most masters, in being able year after year to increase his circle of friends from the ranks of his former pupils, on whose attitude to life and its problems he had a great influence, unconsciously asserted. The friends, colleagues or pupils who took part in the adventurous tours in the Hebrides in a 6-ton yacht, in which he spent a long series of holidays, did not detect a formal pedant in the intrepid navigator, the genial and strict commander of those never to be forgotten voyages.

MATHEMATICAL NOTES.

538. [R. L^1.] *A Formal Geometrical Construction for the Solution of the Sound Ranging Problem.* By G. H. Bryan, Pres.Inst.Ae.E.

Let S_1, S_2, S_3 be the three stations, P the gun of which the report is heard. Knowing the intervals between the times at which it is heard at S_1 and S_2, we know the difference of S_1P and S_2P, and P lies on a known branch of a hyperbola having this difference as its transverse axis and S_1, S_2 as foci. From these the eccentricity, say e_1, is known, and the directrix can be constructed, and if PM_1 be drawn perpendicular on this directrix, we should have $SP=e_1 . PM_1$. Similarly, from the difference of S_1P and S_3P we construct the directrix and find the eccentricity e_2 of a hyperbola with S_1 and S_3 as foci, on which P also lies. If PM_2 is perpendicular on this directrix $S_1P=e_2 . PM_3$. Hence the ratio $PM_1 : PM_2$ is known, and the locus of a point satisfying this condition is a straight line which can be constructed. Having found this line, the point P in which it intersects one of the conics can be constructed by means of the eccentric angle. This method, though illustrative of formal geometrical constructions, would of course be unsuitable for actual working. In gunnery observations it is found more convenient and sufficiently accurate to use the asymptotes instead of the hyperbolas themselves in determining the point to be aimed at.

G. H. Bryan.

539. [X. 10.] *A Curiosity.*

$$\frac{18534}{9267}\times\frac{17469}{5823}=\frac{34182}{5697}.$$

Here the set of digits occurs in each fraction, each digit once and only once.

A. O. P.

540. [J. 2.] If the chances of A and B winning a game at lawn tennis are x and $1-x$, then the chance that A will win a set is

$$x^6+\left\{1+\binom{6}{1}(1-x)+\binom{7}{2}(1-x)^2+\binom{8}{3}(1-x)^3+\binom{9}{4}(1-x)^4+\binom{10}{5}(1-x)^5\right.$$
$$\left.+2\binom{10}{5}(1-x)^6x^2/(1-2x\overline{1-x})\right\}.$$

T. Carleman (per G. H. Hardy).

541. [X. 10.] *The Four Fours.*

Here is an original problem which may prove of interest to some readers of the *Gazette*.

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× × ×)× × × × × × 4(× 4 × ×
        × × ×
        × × 4 ×
        × × × ×
          × × × ×
            × 4 ×
            × × × ×
            × × × ×
```

There are just four ways of filling up the missing figures so as to leave a correctly-worked long division sum (in scale ten), and the question is to find them.

W. E. H. Berwick.

University College, Bangor, North Wales.

REVIEWS.

(1) **Lectures on the Principle of Symmetry and its Application in all Natural Sciences.** By Professor F. M. JAEGER. Pp. xii + 334. 20s. net. 1919. (Cambridge University Press.)

(2) **On Growth and Form.** By D'ARCY WENTWORTH THOMPSON. Pp. xvi + 793. £1 1s. net. 1917. (Cambridge University Press.)

(3) **The Quantitative Method in Biology.** By J. MACLEOD. Pp. xii + 228. 15s. net. 1919. (Longmans, Green & Co.)

An innate love of beauty of form and colour is one of the main influences which have led to the development of the most elementary method of studying natural history by making collections of specimens. The effect of this influence is well demonstrated by the disproportionate amount of attention paid to Lepidoptera as compared with other orders of insects, and the popularity of the shells of the lovely little Diatomaceae apart from their vegetable contents. Collecting naturally leads to classification and the study of distribution, but after a time a desire sets in to know more about the objects, and up till recently this desire has been met by extended observations of the life history and development of individual species, a study which affords ample occupation for research scholars to occupy their time. In this stage the study of form is subservient to that of function. We have now reached a further stage in a reversion to the study of form in the organic world with the object of ascertaining the causes to which it is due. In this development geometrical considerations naturally play an important part.

Now the branch of geometry which is most complete in itself, and which is at the same time most precise in its applicability to such studies, is the Principle of Symmetry, which forms the subject of Prof. Jaeger's treatise. The limitations to the partitioning of space govern equally all forms whether organic or inorganic which occupy space. Hitherto the principles of symmetry have been mainly studied in their applications to crystallography, and it is therefore exceedingly interesting to observe how similar forms are reproduced in pollen grains, inflorescences, diatoms, and above all in the exquisite shells of the *Polycistina.* It is a pity that students of these latter forms confine their attention so much to those occurring in Barbados Earth, for simple beauty of form is also characteristic of species which can be picked out from gatherings on our own shores.

The author does not claim to have produced a treatise on the mathematical theory, but rather an enumeration of the objects which illustrate it. The reader who is not an expert at crystallography may have a little difficulty in interpreting the diagrams of crystals.

The book is nicely got up, and the illustrations are excellent. There is, however, one trifling feature which is rather unsightly, namely, the irregularity in the lines of print which occurs every time the author introduces the fraction $\frac{2\pi}{n}$. Even in some cases $\frac{n}{2}$ upsets the symmetry of the pages.

Prof. D'Arcy Thompson treads on much more dangerous ground in applying dynamical and physical principles to the growth and development of organic forms. He fully justifies his attempts by the happily written preface, or "prefatory note" as he calls it—for D'Arcy Thompson does not descend to the modern practice of spoiling a good English book by introducing the Hun's term "Vorwort," which disfigures so many recent books. There is ample justification for his invading the province of the mathematician. Workers in every branch of science have to do the same, for they have learnt from bitter experience that if they wait for mathematicians to take up their problems they will never get them done. Hence the growing use of graphical diagrams plotted on squared paper as a substitute for rigorous deductions from precisely stated assumptions. The early chapters deal largely with the rate of growth, and in these the graphic method is largely used. In connection with

cell development, the author has to take account of dynamical considerations, such as surface tension as applied to cell walls, in addition to considerations regarding the partitioning of space. The most fascinating chapters, however, are those dealing with the forms of spicules and of foraminifera and other shells. In connection with spiral shells the author seems very fascinated with the properties of the logarithmic spiral. Whether or not it is really worth while to introduce the equation of the curve at all is questionable. The practice followed by the author in this respect does not do any harm, but after all the whole thing would be much better regarded as summed up in elementary geometrical considerations. It all comes to this:

If a spiral growth increases by similar elements in such a way that the dimensions (linear, superficial or volume) of each added element are proportional to the corresponding dimensions of the whole organism, then the spiral is equiangular. And it is a pity that the author does not use the word "equiangular" instead of "logarithmic." Reference is also made to the spiral of Archimedes, in which the radius increases by the same amount in each whorl. On page 505 there is a trifling mathematical inaccuracy in the second line which does not matter at all. But we might point out that the spiral in which each whorl is of constant *thickness* measured normally (not radially from a pole) is an involute of either a circle or some closed oval curve, and something *resembling* this condition may be seen if Prof. Thompson will examine Nummulites split both transversely and perpendicularly to the disc. In subsequent chapters the author reproduces diagrams of bridge girders, which he applies to the study of limbs and bones, and in the last chapter on "The Comparison of Related Forms," he applies the general method of transforming one space diagram into another by transformations which will shock the cut and dried mathematician by being non-conformal, non-orthogonal, and what is better still, not disfigured by algebraic formulae.

The same cannot be said of Dr. M'Leod's book on "The Quantitative Method in Biology." In avoiding the objections which attach to mere collecting specimens for collecting's sake we have here an illustration of the danger which awaits us at the opposite extreme, namely, the reduction of everything to mere collections of *symbols*. If it is necessary to introduce algebra into the study of biology, let us hope that some means may be discovered of rendering the subject less cut and dried than is the case in this book. Not content with introducing collections of 21, 27 or even more unsightly things like A^2a^2, aa, bb, b^2 and other objects of the "piarar" class, almost every other paragraph has the old familiar heading EXAMPLE, which we get so heartily sick of the sight of when we begin to study mathematics in schoolboy days. Surely the charm of biology should be that its study ought to make one feel that there is something to live for in this world, and that life is not all made up of working "examples" and answering "revision papers." Then, again, as regards the so-called "biometric methods" in comparing the dimensions of different specimens, it is very difficult to select suitable standard measurements on which to base the comparison. Even in such a different application as the classification of aeroplanes of different sizes and shapes it is difficult to decide what linear measurement is to be taken to represent the size of the wings when the span as well as the chord may be different in different machines. It is quite clear that if all this algebra is to be of any good it must be applied to a large number of different measurements of the same thing, and such books will have to be increased n fold or more.

In may, however, be allowed that the subject admits of industrial application in connection with the improvement of useful species by artificial selection; if so, we hope it may be possible to make the study rather more attractive than it is at present. Even the pages of the book have not been sufficiently guillotined, and insist on opening at certain places and not at others.

On the other hand, the produce of our university system includes a considerable leaven of post-graduate students who seem capable of appreciating and even of revelling in the formal methods and pedantic language of this book, and if it should enable these workers to benefit society by producing applications of proved ability, it will not have been written in vain.

G. H. BRYAN.

Introduction to Mathematical Philosophy. By Bertrand Russell. Pp. viii, 208. 10s. 6d. net. 1919. (London: Allen and Unwin.)

A clear and well-written account, in which the symbols of what Mr. Russell calls "mathematical logic" are not used, of some of the mathematical and logical work of Cantor, Dedekind, Peano, Frege, Russell, Whitehead, and a few others on the exact definition or treatment of such terms as "number," "infinity," "order," "limit," "continuity," and "class," and the deduction of some properties of the ideas which these terms denote. Thus the title seems misleading. The exposition is hurried and often degenerates into bald repetition of Mr. Russell's views (*e.g.* pp. 167 ff.). Often, too, they are unconvincing (*e.g.* pp. 135 ff.), dogmatic (*e.g.* pp. 53, 117), or superficial (*e.g.* pp. 71, 79, 88, 97, 107, 142, 155 ff.). A misleading account is given (p. 107) of the discovery of the fact that the "continuity" of a function is a purely ordinal notion, and a "function" is (p. 46) arbitrarily limited to be one-valued.

Philip E. B. Jourdain.

A First Course in the Calculus. Part I. By W. P. Milne, D.Sc., and G. J. B. Westcott, M.A. Pp. i-vii+196. 3s. 6d. 1919. (Bell & Sons.)

Only a few years have passed since, in an introductory chapter of one of the most stimulating and original mathematical text-books that has appeared in the English Language, the author found it necessary to remark:—"When we consider the position of the differential and integral calculus, we have to protest against a tradition which forbids all but exceptional pupils to become acquainted with the most powerful and attractive of mathematical methods." To-day, generally speaking, that tradition is dead: and the results of disregarding it have been so surprising in their success that no fear can be entertained of its revival. At the same time, it is open to doubt whether sufficient experience has accumulated to determine precisely what part the calculus should play in the curriculum of the non-specialist. There is the obvious danger that it may become simply one more subject in which pupils will be expected to reach a defined standard; that a stereotyped course will be developed which examinations will foster; and that the element of choice and flexibility which is at present such a valuable asset may disappear.

As long as Examining Bodies withhold their hands, teachers will be left free to decide what they will include and what omit: and under such conditions a strong case can be made out for the miscellaneous type of text-book which exhibits a wide variety of analysis and applications rather than the separate-subject text-book, as far as non-specialists are concerned. External conditions are no doubt mainly responsible for the water-tight compartment method in British education. And it is hard to deny that greater facility in technique and capacity to tackle a normal straight-forward example is produced by the single-subject text-book where attention is focussed on a central purpose: progress is measured by concrete landmarks (maxima and minima to-day, asymptotes next week), which induces in the teacher a sense of satisfaction. But steady advance along a definite path, essential as it is for the specialist, necessitates some sacrifice of breadth of view, which, when mathematical hours are few as in the case of the non-specialist, becomes so serious that many will prefer a discursive range and a multiplicity of idea. For those, however, who are reading mathematics for engineering or some similar purpose, the custom of "one subject, one text-book" will persist. To such, we can strongly recommend Messrs. Milne and Westcott's small volume as a first course in the Calculus. It is clear and compact: in about 150 pages, it gives the reader a capital bird's-eye view of the subject and its ramifications. The treatment is restricted to powers of x, but applications are made to pneumatics, geometry, areas, volumes, centres of gravity, pressure and moments of inertia. The bookwork is set out very briefly, and in our opinion rightly so, for pupils are easily confused and depressed by overmuch printed talk. Occasionally, however, the authors seem rather too abrupt. We should like to see rather more explanation of the phrase "in the limit," and of the formula $\frac{dz}{dx}=\frac{dz}{dy}\cdot\frac{dy}{dx}$; and some of the examples of differential equations in chapter vi., drawn from work which the reader is unable to under-

stand, seem to us to make ineffective illustrations. But these are trivial points: the plan of the book and its general execution deserve high praise, no less than the admirable quality of the examples. We shall await with interest the promised publication of a Second Part. C. V. D.

Projective Vector Algebra. By L. Silberstein. Pp. vii+78. 7s. 6d. net. 1919. (Bell and Sons.)

In this tract of 76 pages, Dr. Silberstein, with his accustomed lucidity, develops an interesting variety of vector algebra. Take any two coinitial vectors OX, OY represented by $\mathbf{X}$, $\mathbf{Y}$; produce them to any convenient *terminal* points T_xT_y; draw XT_y, YT_x, crossing in A; this point A is a unique point in the "restricted region" OT_1T_2 and the vector OA or $\mathbf{A}$ a unique vector. Then, by definition, $\mathbf{A}=\mathbf{X}+\mathbf{Y}$. Take now any third vector $\mathbf{Z}=OZ$, and let its terminal point be T_z. $T_xT_yT_z$ will form a planes; also somewhere in T_xT_y there will be the terminal point T_A obtained by producing OA till it meets the line $T_x'T_y$. AT_z and ZT_A will meet in a definite point B, the end of the vector, OB or $\mathbf{B}$. Then $\mathbf{B}=(\mathbf{X}+\mathbf{Y})+\mathbf{Z}$. It is then shown that this vector $\mathbf{B}$ is also $=(\mathbf{Y}+\mathbf{Z})+\mathbf{X}=(\mathbf{Z}+\mathbf{X})+\mathbf{Y}$. On these definitions and demonstrations the new vector algebra is based. The author proceeds to consider the meaning of equal vectors, of multiples of vectors, and of sub-multiples of vectors, and then gives examples showing the applicability of the vector algebra to the whole field of projective geometry. One important section, for example, deals with Pascal's Theorem, which is proved in a remarkably simple way. Conic Ranges and Pencils form the subject matter of the last section covering 14 pages. In an Appendix two points of interest are dealt with. The one is the treatment of angles between vectors, equal angles being defined as those between two pairs of vectors, when each vector of one pair is *equal* (in the meaning of the algebra) to the corresponding vector of the other pair. It is then proved that the non-metrical sum of the angles of a triangle is equal to a straight angle, a generalisation of the well-known Euclidean theorem. The other subject discussed in the Appendix is the Linear Vector Operator, which is developed along familiar vectorial lines. There is no doubt that once the significance of the fundamental definitions is grasped and the mind is cleared of obsessions of Euclidean geometry, the student will find in this elegant vector algebra a ready instrument for research in the domain of projective geometry. C. G. K.

THE YORKSHIRE BRANCH.

A preliminary meeting was held on Feb. 21, in the Board Room, University House, Leeds, to consider the formation of a Yorkshire Branch of the Mathematical Association. It was attended by nearly a hundred men and women teachers in Secondary Schools, Technical Colleges, and Universities. Prof. W. P. Milne, who was voted to the chair, explained the desirability of forming such a body in order to further sound mathematical teaching throughout the county. The greatest enthusiasm was displayed, and it was resolved unanimously to form a branch. The following officers and committee were then elected: Chairman, Professor W. P. Milne; Treasurer, Miss I. E. Cameron, Head Mathematical Mistress, Leeds Girls' High School; Secretary, Rev. A. V. Billen, Senior Mathematical Master, Leeds Grammar School; and Executive Committee, Mr. Beard (Wakefield), Mr. Blacklock (Rotherham), Dr. Brodetsky (Leeds), Miss Cull (Wakefield), Mr. Gilliam (Leeds), Miss Greene (Bradford), Dr. Clement Jones (Bradford), Professor Leahy (Sheffield), Mrs. Pochin (Leeds), Mr. Sadler (Dewsbury), and Mr. Stewart (Sheffield).

A discussion then took place on the introduction of Logarithms and Numerical Trigonometry into the regular syllabus of the University matriculations, and after a vigorous debate a sub-committee was appointed to prepare detailed proposals. Dr. Brodetsky then gave a short account of Einstein's Theory.

The Mathematical Association will hold its general meeting at Leeds in May, when a Yorkshire greeting will be extended to its parent by the infant branch, already a lusty bantling.

REPORT OF SYDNEY BRANCH FOR 1919.

During the year only two meetings were held, one in October, at which Mr. Wellish read a paper on "The Teaching of Mechanics"; the other, the Annual Meeting in November, at which there was a discussion on the Leaving Certificate Examination papers of this year.

The influenza epidemic prevented the holding of meetings in the earlier part of the year. The distribution of the *Gazette* has proceeded satisfactorily. There are now 19 members and 50 associates belonging to this branch. In addition, two school libraries are subscribing to the Association for the purpose of obtaining the *Gazette*.

At the Annual Meeting, the following office-bearers were elected for the ensuing year: President, Prof. H. S. Carslaw; Joint Hon. Secretaries, H. J. Meldrum and Miss F. Cohen; Hon. Treasurer, Dr. E. F. Simonds.

NOTICES.

The International Congress of Mathematicians. The Congress will be opened at Strasbourg, on Sept. 22, 1920. Authors of papers should apply for information to M. Koenigs, 96 Boulevard Raspail, Paris. For other information application may be made to M. Villat, 11 rue du Maréchal-Pétain, Strasbourg, or to M. Galbrun, 114 avenue Émile-Deschanel, Paris.

Prof. R. G. Diena, via Zecca Vecchia 4, Milano, will be grateful to any reader of the *Gazette* who can send him a list of books, ancient or modern, dealing with Perpetual Motion.

The Council of the Mathematical Association has sanctioned the free use of our columns, by **Members only**, for notices of Scientific Books and Journals for sale or exchange. Such notices and replies thereto should be addressed to The Editor, *The Mathematical Gazette*.

Reprints of many pamphlets, memoirs, etc., by the late P. E. B. Jourdain, may be obtained at an average price of 9d. each. For list, apply Mrs. Jourdain, c/o Editor.

THE LIBRARY.

Change of Address.

The Library is now at 9 Brunswick Square, W.C., the new premises of the Teachers' Guild.

The Librarian acknowledges, with thanks, the gift, by Sir George Greenhill, of 4 more volumes of the *Bulletin of the American Mathematical Society*, making complete Vols. I. to XXV.; also of Vols. I., II., IV., XI. to XXII. of the *Jahresbericht der Deutschen Mathematiker Vereinigung*; also the gift, by Miss M. E. Rickett, of 41 back numbers of the *Gazette*.

Scarce Back Numbers.

Reserves are kept of A.I.G.T. Reports and Gazettes, and, from time to time, orders come for sets of these. We are now unable to fulfil such orders for want of certain back numbers, which the Librarian will be glad to buy from any member who can spare them, or to exchange other back numbers for them:

Gazette No. 8 (very important).
A.I.G.T. Report No. 11 (very important).
A.I.G.T. Reports, Nos. 10, 12.

GLASGOW: PRINTED AT THE UNIVERSITY PRESS BY ROBERT MACLEHOSE AND CO. LTD.

BOOKS RECEIVED, CONTENTS OF JOURNALS, ETC.

March, 1920.

Académie Royale de Belgique. Programme des Concours Annuels : Rapport succinct sur l'Etat du Palais des Académies après le depart des Allemands : Fondations Académiques. 1919.

An Elementary Course of Infinitesimal Calculus. By H. LAMB. Revised Edition. Pp. xiv + 530. 20s. net. 1919. (Cambridge Univ. Press.)

Elementary Calculus. By C. H. P. MAYO. Pp. vi + 345. 10s. 1919. (Rivingtons.)

Elements of Physics. By R. A. HOUSTON. Pp. viii + 221. 6s. net. 1919. (Longmans, Green.)

Elements of Vector Algebra. By L. SILBERSTEIN. Pp. 42. 5s. net. 1919. (Longmans, Green.)

Intermédiaire des Mathématiciens. Nos. 9-10. Sept.-Oct. 1919.

Mensuration for Marine and Mechanical Engineers. By J. W. ANGLES. Pp. xxvii + 162. 5s. net. 1919. (Longmans, Green.)

Principles and Practice of Electrical Testing. By R. G. ALLEN. Pp. 362. 18s. net. 1919. (Longmans, Green.)

The Principles of Natural Knowledge. By A. N. WHITEHEAD. Pp. xii + 200. 12s. 6d. net. 1919. (Cambridge Univ. Press.)

Researches in Physical Optics. Part II. Resonance Radiation and Resonance Spectra. By R. W. WOOD. Pp. viii + 184. 1.50 dols. 1919. (Columbia University Press, New York.)

School Statics. Part I. By W. G. BORCHARDT. Pp. viii + 266. 6s. 1919. (Rivingtons.)

American Journal of Mathematics.

July 1919.

Invariants of Differential Geometry by the Use of Vector Forms. Pp. 165-182. C. D. RICE. *On Certain Saltus Equations.* Pp. 183-190. H. BLUMBERG. *Investigations on the Plane Quartic.* Pp. 191-211. T. COHEN. *On Surfaces containing Two Pencils of Cubic Curves.* Pp. 212-224. C. H. SISAM. *Modular Invariants of a Quadratic Form for a Prime Power Modulus.* Pp. 225-242. J. E. M'ATEE.

The American Mathematical Monthly.

June 1919.

DISCUSSIONS : *Trigonometrical Functions—of What?* Pp. 239-244. W. B. CARVER. Mathematical Clubs. *Flatlanders—A Mathematical Play in one Act.* Pp. 264-267.

Sept. 1919.

A Theory and Generalization of the Circular and Hyperbolic Functions. Pp. 280-288. A. F. FRUMVELLER. *A Proof of a Theorem of Compound Probabilities.* Pp. 288-290. W. BROGGI. *Bits of History about two Common Mathematical Terms* [Indicator, Simple Group]. Pp. 290-291. G. A. MILLER.

DISCUSSIONS : *The Complex Quantity in Algebra.* Pp. 295-297. W. WOOLSEY JOHNSON. *On Teaching of Logarithms.* R. B. M'CLENNON.

Oct. 1919.

The Growth of the Solar System. Pp. 326-332. W. D. MACMILLAN. *Cuspidal Rosettes.* Pp. 332-341. W. F. RIGGE. *On the Teaching of the First Course in the Calculus.* Pp. 342-344. K. L. RIETZ. *The Sign of the Distance in Analytical Geometry.* Pp. 344-350. A. A. BENNETT. *Probabilities in the Game of "Shooting Craps."* Pp. 351-352. B. H. BROWN. *Note on the Number of Solutions of Linear Indeterminate Equations.* Pp. 365-366. D. N. LEHMER.

Nov. 1919.

On the Quadrature of the Parabola. Pp. 388-390. R. E. MORITZ. *Geometrical Construction of the Roots of a Cubic, and Inscription of a Regular Heptagon in a Circle.* Pp. 390-392. L. B. HALDEMAN. *On the Summation of Certain Series.* Pp. 392-395. O. SCHMIEDEL. *An Erroneous Rule for Finding an Hypotenuse, with a Corollary.* Pp. 395-396. M. W. JACOBS.

Annals of Mathematics.

June, 1919.

Relations between Abstract Group Properties and Substitution Groups. Pp. 229-231. G. A. MILLER. *The Complete Quadrilateral.* Pp. 232-261. J. W. CLAWSON. *Triply Conjugate Systems with Equal Point Invariants.* Pp. 262-273. L. P. EISENHART. *On a System of Linear Partial Differential Equations of the Hyperbolic Type.* Pp. 274-278. T. H. GRONWALL. *Some Properties of Circles and Related Conics.* Pp. 279-280. J. H. WEAVER. *Integrals in an Infinite Number of Dimensions.* Pp. 281-288. *On the Differentiability of the Solution of a Differential Equation with Respect to a Parameter.* Pp. 289-291. J. F. RITT. *Note on the Derivatives with Respect to a Parameter of the Solutions of a System of Differential Equations.* Pp. 292-296. T. H. GRONWALL. *On Quaternions and their Generalization, and the History of the Eight-Square Theorem. Addenda.* Pp. 297. L. E. DICKSON.

Sept. 1919.

Investigation of a Class of Fundamental Inequalities in the Theory of Analytic Functions. Pp. 1-29. J. L. W. V. JENSEN. *Functions of Limited Variation in an Infinite Number of Dimensions.* Pp. 30-38. P. J. DANIELL. *A new Sequence of Integral Tests for the Convergence and Divergence of Infinite Series.* Pp. 39-60. R. W. BRINK. *Calculation of the Complex Zeros of the Function P(z) complementary to the Incomplete Gamma Function.* Pp. 61-63. P. FRANKLIN. *Total Differentiability.* Pp. 64-72. E. J. TOWNSEND.

Anuario para el Año 1918. No. 9. Feb. 1918. (Univ. Nacional de La Plata. Facultad de Ciencias Físicas, Matemáticas y Astronómicas.)

Bulletin of the Calcutta Mathematical Society. (University Press, Calcutta).

IX. 1.

On the Normal Derivate of the Newtonian Potential due to a Surface Distribution having a Discontinuity of the Second Kind. Pp. 1-11. G. PRASAD. *On the Angle-Concept in n-Dimensional Geometry.* Pp. 11-18. S. M. GANGULY. *A Note on Current Views of Certain Operations through the Fourth Dimension.* Pp. 19-21. S. MOOKERJEE. *On Rotations about Concurrent Axes and the Polar of a Spherical Polygon.* Pp. 23-42. C. E. CULLIS. *On the Forced Vibrations of a Heterogeneous String.* Pp. 43-58. S. BANERJI.

IX. 2.

On the Figures of Equilibrium of two Rotating Masses of Fluid for the Exponential e^{-kr}/r. Part I. pp. 59-70. A. DATTA. *Fourier's Series and its Influence on some of the Developments of Mathematical Analysis.* Pp. 71-84. R. A. C. BOSE BAHADUR. *On the Numerical Calculations of the Roots of the Equations* $P_n^m(\mu)=0$ *and* $\frac{d}{d\mu}P_n^m(\mu)=0$ *regarded as Equations in n.* Pp. 85 95. B. PAL. *On some new Theorems in the Geometry of Masses.* Pp. 97-107. S. DHAR. *On the Electric Resistance of a Conducting Spheroid with given Electrodes.* Pp. 109-114. S. KAR.

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On Surface Waves and Tidal Waves near a Promontory. Pp. 1-10. S. BANERJI. *On the Potentials of Uniform and Heterogeneous Elliptic Cylinders at an External Point.* Pp. 11-27. N. SEN. *Notes on Inversion.* Pp. 29-34. T. BHATTACHARYYA. *On the Use of Ritz's Method for Finding the Vibration Frequencies of Heterogeneous Strings and Membranes.* Pp. 35-42. N. K. MAJUMDAR. *On the Steady Motion of a Viscous Fluid due to the Rotation of two Rigid Bodies about Arbitrary Axes.* Pp. 43-61. B. DUTT.

X. 2.

New Methods in the Geometry of a Plane Arc. Pp. 65-72. S. MUKHOPADHYAYA. *Origin of the Indian Cyclic Method for the Solution of* $Nx^2+1=y^2$. Pp. 73-80. P. C. SENGUPTA. *On the Motion of an Ellipsoid of Revolution in a Viscous Fluid in the Light of Prof. Oseen's Objection to Stokes's Treatment of the Case of the Sphere.* Pp. 81-94. B. PAL. *On a Class of Ellipsoidal Harmonics and a Method of solving the Wave Equation in Ellipsoidal Coordinates.* Pp. 95-104. S. BANERJI. *Some Cases of Tidal Oscillations in Canals of Variable Section.* Pp. 105-116. S. DASGUPTA. *The Stress Equations of Equilibrium.* Pp. 117-121. S. BASU.

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April 1919.

Principio de los trabajos virtuales. Applicaciones. Pp. 9-71. J. L. BIMBI.

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June 1919.

Les noms et les choses. Remarques sur la nomenclature mathématique. Pp. 237-244. GINO LORIA. *Les Origines d'un problème inédit de E. Torricelli.* Pp. 245-268. E. TURRIÈRE. *Remarques sur l'intégrale* $\int uv\,dx$. Pp. 268-270. M. PETROVITCH. *Combinaisons déterminantes.* Pp. 271-276. J. HELMIS. *Sur la détermination et quelques propriétés des lignes élastiques.* Pp. 276-285. M. ZACK. *Sur les équations transcendantes qui se présentent dans la théorie des tiges élastiques.* Pp. 286-292. M. PASCHOUD.

Oct. 1919.

Sur une transformation élémentaire [$y = f(x)/g(x)$ where $f(x)$ and $g(x)$ are polynomials of second degree] *et sur quelques intégrales définies et indéfinies.* Pp. 317-337. C. CAILLER. *Sur l'intégrale* $n!\int_0^h [h^n e^{-hx/(1-h)}/(1-h)]\,dh$. Pp. 338-346. F. VANCY and M. PASCHOUD. *Extension de la notion de Jacobien.* Pp. 349-354. M. STUYVAERT. *Sur la représentation proportionelle en matière électorale.* Pp. 355-379. G. PÓLYA.

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Aug. 1919.

On the Cartesian Oval. Pp. 123-144. V. RAMASWAMI AIYAR. *A Complaint against Text-Books.* Pp. 145-154. W. A. GARSTIN.

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A Problem of Diophantine Approximation. Pp. 162-166. G. H. HARDY. *A General Theorem relating to the Cartesian Oval.* Pp. 167-172. A. C. L. WILKINSON. *Multiplication of Infinite Integrals.* Pp. 173-180. K. B. MADHAVA. *A Proof of Bertrand's Postulate.* [His conjecture that if $x \geq 1$ there is at least one prime p, such that $x < p \leq 2x$.] Pp. 181-182. S. RAMANUJAM.

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Ed. by Messrs. KABA, KURODA, WATANABE, NAGAO, and FURUKAWA.

Vol. I. No. 1. April 1919. (In Japanese.)

Memoria correspondiente a 1917. (Publicaciones de la Facultad de Ciencias físicas, matemáticas, y astronómicas. Universidad nacional de la Plata.)

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Sur une application élémentaire d'une methode générale donnant les équations du mouvement d'un système. Pp. 121-131. P. APPELL. *Sur les centres de courbure des lignes décrites par les points d'une figure plane mobile dans son plan.* Pp. 131-134. M. D'OCAGNE. *Sur l'extraction, à une unité près, de la racine m*e *d'un nombre quelconque à l'aide des logarithmes.* Pp. 134-137. M. P. DELENS. *Sur l'orthopôle.* Pp. 137-140. R. GOORMAGHTIGH. *Sur quelques intégrales trigonométriques.* Pp. 140-145. M. F. EGAN.

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Réduction à une forme normale d'un système d'équations différentielles simultanées linéaires à coefficients constants. Pp. 201-209. H. VOGT. *Le théorème de Feuerbach dans les cubiques.* Pp. 210-213. M. MALGOUZOU. *Démonstration du théorème de Chasles sur les arcs de lemniscate.* Pp. 213-215. F. BALITRAND. *Sur les conditions pour qu'une fonction $P(x, y)+iQ(x, y)$ soit monogène.* Pp. 215-219. *Groupes de points sur l'hyperbole équilatère.* Pp. 220-228. J. SER.

July 1919.

Sur quelques courbes associées à une classe d'hélices cylindriques. Pp. 241-248. M. EGAN.

Aug. 1919.

Théorème général sur les équations algébriques. Pp. 281-284. M. PETROVITCH. *Sur l'identité de Bezout* [$AU+BV=1$; A and B integral polynomials in x, of degrees m and p respectively, U_1 and V_1 polynomials of degrees $p-1$ and $m-1$ at most, such that $AU_1+BV_1=1$; and $U=U_1+Bf(x)$, $V=V_1-Af(x)$]. Pp. 284-297. B. GAMBIER.

Sept. 1919.

Démonstration du théorème fondamental de la théorie des équations algébriques. Pp. 321-324 L. POMEY. *Sur le rapport anharmonique.* Pp. 325-329. A. AURIC. *Sur les courbes à axe orthoptique et les courbes de direction.* Pp. 329-338. M. d'OCAGNE. *Sur les systèmes des cercles ayant pour diamètres les segments des tangentes à une ellipse compris entre les tangentes à cette ellipse en ses sommets opposés.* Pp. 338-342. P. DE PLESSIS.

Proceedings of the London Mathematical Society.

July 26, 1919.

The Simultaneous System of Two Quadratic Quaternary Forms. Pp. 81-93. H. W. TURNBULL. *The Nature of a Moving Electric Charge and its Lines of Electric Force.* Pp. 95-135. H. BATEMAN. *A New Method of describing a Three-Bar Curve.* Pp. 136-140. COL. R. L. HIPPISLEY. *On the Connexion between Legendre Series and Fourier Series.* Pp. 141→. W. H. YOUNG.

Sept. 5, 1919.

On the Connexion between Legendre Series and Fourier Series (cont.). Pp. 161-162. W. H. YOUNG. *On the Series of Bessel Functions.* Pp. 163-200. W. H. YOUNG. *Additional Note on Two Problems in the Analytic Theory of Numbers.* Pp. 201-204. G. H. HARDY. *Abel's Theorem and its Converse.* Pp. 205-235. G. H. HARDY and J. E. LITTLEWOOD. *On the Continued Fraction connected with the Hypergeometric Equation.* Pp. 236→. E. L. INCE.

Proceedings of the Royal Society. Series A. Vol. 95. No. 673.

July 1919.

An Apparatus for the Direct Determination of Accelerations. Pp. 492-507. PRINCE B. GALITZIN. *The Two-Dimensional Motion of a Plane Lamina in a Resisting Medium.* Pp. 516-532. S. BRODETSKY. *The Transmission of Electric Waves round the Earth.* Pp. 546-563. G. N. WATSON.

Rendiconti del Circolo Matematico di Palermo.

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Revista Matemática Hispano-Americana. Edited by J. REY PASTOR. Vol. I. No. 6.

June 1919.

Sobre las trasformaciones puntuales. Pp. 169-171. J. HADAMARD. *El problema de los tres cuerpos.* Pp. 172-177. J. M. PLANS. *Las obras matemáticas de Juan Caramuel.* Pp. 178-189. D. F. DIEGUEZ. *La sucesión de Fibonacci.* Pp. 190-193. F. VERA.

Sept. 1919.

El tratado de la Logarítmica de Juan Caramuel. Pp. 203-212. D. D. F. DIÉGUEZ. *Puntos notables del triangulo.* Pp. 213-217. P. M. G. QUIJANO. *Conjuntos de funciones y de subconjuntos.* Pp. 218-223. J. RAY PASTOR.

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D. A. da Silva. Pp. 237-243. F. GOMES TEIXEIRA. *Puntos notables del triángulo.* Pp. 244-251. P. M. G. QUIJANO. *Algunas nociones de Análisis Situs.* Pp. 252-258. O. F. BAÑOS. *La Sucesión de Fibonacci.* Pp. 259-264. *Volumen del prismatoïde.* Pp. 293-296. L. CATALA.

School Science and Mathematics.

Oct. 1919.

What Graphical and Statistical Material should be included in the Ninth Year Course? Pp. 595-598.

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What is Practical Mathematics? D. H. MOSKOWITZ. Pp. 827-829. *Marginal Notes on Cajori's History of Mathematics.* Pp. 830-835. G. A. MILLER. *The Socialised Recitation in Mathematics.* Pp. 844-848. F. E. BUSS.

The School Science Review. June 1919. Vol. 1. No. 1. 2s. net. (Murray.)

Scientia. Edited by E. RIGNANO.

I. VI. 1919.

Le matematiche in Ispagna, ieri ed oggi. II. *I matematici moderni.*

I. XII. 1919.

Il posto di Leonardo nella storia delle Scienze. Pp. 437-448. A. FAVARO.

Wiskundig Tijdschrift.

Zestiende Jaargang 2.

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